Remarks on exhaustification and embedded free choice

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Abstract
Some sentences that contain disjunctions imply that their disjunct-alternatives are false, while others imply that they are true. Recent work on scalar implicature has been guided by the behavior of such constructions. In this paper I consider examples in which free choice disjunctions (phrases of the form *allowed to A or B*) appear in various intensional contexts. I discuss the significance of the findings in light of current views of exhaustification.

1 Introduction

Much of the recent research on scalar implicature has been guided by sentences that contain disjunctions, specifically by the inferences that they license with respect to what we may call their 'disjunct-alternatives'. A disjunct-alternative S' to a (disjunctive) sentence S is one where the disjunction $(p \vee q)$ in S is replaced with either p or q in S'. In some cases a disjunctive sentence suggests that its disjunct-alternatives are true; in others it suggests that they are false; and in yet others it suggests that their value is unknown. Examples of the form $\Diamond(p \vee q)$ are of the first kind, e.g., (1); examples of the form $\Box(p \vee q)$ are of the second (2); and unembedded disjunctions are of the third (3).

(1) Chris is allowed to eat salad or soup. $\Diamond(p \vee q)$
  a. $\rightsquigarrow$ Chris is allowed to eat salad $\Diamond p$
  b. $\rightsquigarrow$ Chris is allowed to eat soup $\Diamond q$

(2) Chris is required to eat salad or soup. $\Box(p \vee q)$
  a. $\rightsquigarrow$ Chris is not required to eat salad $\neg\Box p$
  b. $\rightsquigarrow$ Chris is not required to eat soup $\neg\Box q$

(3) Chris ate salad or soup. $(p \vee q)$

a. $\rightsquigarrow$ Speaker is not certain that Chris ate salad $\neg K(p)$

b. $\rightsquigarrow$ Speaker is not certain that Chris ate soup $\neg K(q)$

Many see the inferences in (1), and possibly in (2), to come from the composition of the literal meanings of disjunction and the operators that accompany it. Accounts of this kind depart from the tradition that treats disjunction as a Boolean operator, and from views of modals as simple propositional operators.[1] Others have argued that no such revision is necessary, and that the inferences of interest result from an external strengthening mechanism—external, that is, to the (Boolean) meaning of disjunction and the meanings of the accompanying operators.

My points in this paper concern views of the second kind. I pay attention to two datasets. The first extends (2), consisting (roughly speaking) of sentences of the form $\Box(p \vee q)$. These are argued to imply the falsity of their disjunct-alternatives, as is shown for (2) above. The second dataset consists of sentences in which constructions like (1), of the form $\Diamond(p \vee q)$, are embedded under other operators. Some of these operators are the same as the ones that appear in the first dataset, giving us sentences of the form $\Box\Diamond(p \vee q)$. In other cases the embedding operators carry existential quantificational force, giving us sentences of the form $\exists x(\Diamond(Px \vee Qx))$ and the form $\Diamond\Diamond(p \vee q)$. I claim that each sentence type in the second dataset allows a reading that implies "embedded free choice": for sentences of the form $\Box\Diamond(p \vee q)$, this is the inference $\Box(\Diamond p \;\&\; \Diamond q)$, and for sentences of the forms $\exists x(\Diamond(Px \vee Qx))$ and $\Diamond\Diamond(p \vee q)$, it is the inference $\exists x(\Diamond Px \;\&\; \Diamond Qx))$ and $\Diamond(\Diamond p \;\&\; \Diamond q)$, respectively.

When taken together, the data present a theoretical challenge that I discuss, first, in the context of Del Pinal et al.'s (2022) presuppositional view of exhaustification, and second, in the context of Bar-Lev and Fox's (2020) proposal to derive similar inference from global exhaustification. I do not develop a knock-down argument for either of these two views over the other. Instead I simply take note of the assumptions that one needs to adopt (alongside either proposal) to capture the empirical findings.

I begin with the data and a summary of the relevant empirical goals (§2). I then review the basics of exhaustification, and the notions of innocent exclusion and innocent inclusion (§3). In Sect. 4, I revisit the empirical goals from Sect. 2, given the concepts reviewed in Sect. 3. In Sect. 5, I turn to presuppositional exhaustification, and in Sect. 6 to Bar-Lev and Fox's account of related data.

2 The data

2.1 Dataset 1: disjunction under $\Box$

I use the symbol $\Box$ loosely to represent universal(-like) intensional operators, that is, operators that intuitively produce inconsistent results when applied to a propositional

[1] Aloni (2007) and Goldstein (2019) propose this kind of view for cases like (1); Simons's (2005) semantic account applies to (1) and (2); and Geurts (2005) attempts to account for all of (1)-(3) with a modal semantics of disjunction. See Meyer (2020) for a recent review.

argument and simultaneously to its negation. Universal quantifiers, though not intensional, satisfy this criterion, as do the modals *need*, *have to*, *should*, and the attitude verbs *believe*, *think*, and *want*.[2] These are to be contrasted with what we may take to be their duals, which are not always associated with simple lexical items (*be open to the possibility that*, *be okay with*, etc.).

With these assumed instantiations of $\Box$, we find quite robustly that they license distribution inferences; in abstract terms, from a premise of the form $\Box(p \vee q)$ speakers typically draw the inference $\neg\Box p \ \& \ \neg\Box q$.[3] For example, (4) is infelicitous in a context where Bill has to take Syntax 1—the sentence suggests quite strongly that either course would do.

(4) Bill needs to take Syntax 1 or Semantics 1.

We find similar judgments for *believe* and *want*. (5) implies that Bill is not committed to the belief that I play the piano, nor to the belief that I play the electric organ. And (6) implies that my supervisor's desires would be met if I read one of *On Denoting*/*On Referring* but not the other; neither paper is such that my supervisor wants me to read it.

(5) Bill believes/thinks that I play the piano or the electric organ.

(6) My supervisor wants me to read *On Denoting* or *On Referring*.

I take this to show that premises of the form $\Box(p \vee q)$ license the inferences $\neg\Box p \ \& \ \neg\Box q$, where $\Box$ for our purposes is any of *believe*, *want*, or *need*. From this point forward, I adopt standard terminology and call this inference "distribution". Now, if we assume that distribution is a scalar implicature, we derive the following desideratum for any theory of scalar implicature calculation. In (7) this is framed in terms of exhaustification.

(7) **Distribution (DIST)**:
$\mathrm{Exh}(\Box(p \vee q)) \vDash \neg\Box p \ \& \ \neg\Box q$

Deriving this result is fairly straightforward both on a standard definition of Exh, as long as we assume that $\Box(p \vee q)$ has $\Box p$, $\Box q$ as formal alternatives. I say more about this later.

2.2 Dataset 2: embedded free choice

The background to my second dataset, and ultimately to my conclusions from it, is the idea that free choice inferences (FC) also result from exhaustification. (8), repeated from (1), illustrates the inference.

(8) Chris is allowed to eat salad or soup. $\Diamond(p \vee q)$

[2] I assume that it is contradictory for attitude holders to have inconsistent beliefs/desires, and for contextually-determined ordering sources to entail inconsistent propositions.

[3] Ramotowska et al. (2022) report experimental results that suggest otherwise; they find in their experimental setting that participants accept similar disjunctive sentences in scenarios where one of the two disjuncts is required while the other is permitted. I found this report in the final stages of preparing this manuscript, so I cannot give it as much thought as I would like. I return to it in Sect. 6.

a. $\rightsquigarrow$ Chris is allowed to eat salad $\Diamond p$

b. $\rightsquigarrow$ Chris is allowed to eat soup $\Diamond q$

It is not my intention here to defend the idea that FC comes from exhaustification. My more modest goal is to make certain points about the consequences of making this assumption. So I take it as a given that Exh must be defined in a way that generates FC from a prejacent of the form $\Diamond(p \vee q)$:

(9) **Free choice (FC)**:
$\text{Exh}(\Diamond(p \vee q)) \vDash \Diamond p \ \&\ \Diamond q$

Fox (2007) derives FC from recursive exhaustification, and Bar-Lev and Fox (2020) derive it from adding inclusion to the definition of Exh. The details are reviewed in Sect. 3.

2.2.1 Dataset 2a: free choice under $\Box$

Now consider cases where constructions like (8), of the form $\Diamond(p \vee q)$, are embedded under $\Box$, for example (10).

(10) a. Chris believes that Kim is allowed to eat salad or soup.

b. Chris wants to allow Kim to eat salad or soup.

c. Chris needs to allow Kim to eat salad or soup.

(10) has a prominent reading on which FC is calculated in the scope of the embedding operator (*believe*, *want*, *need*). On its own, this finding is easy to explain if we take it that Exh, which by assumption produces FC in (8), can be embedded under *believe, want, need*. If this is right, then it must be that (10a-c) have something like (11) as their LF.

(11) $\Box(\text{Exh}\Diamond(p \vee q))$

But there are similar examples to (10) that also license embedded FC, but that are not as easily explained by embedded exhaustification. Consider (12a,b).

(12) a. Chris needs to allow Kim to eat salad or soup, but I don't.

b. Only Chris needs to allow Kim to eat salad or soup.

It is possible to understand (12a) as saying that Chris needs to give Kim permission to eat salad and permission to eat soup ($\Box_{Chris}\Diamond p$, $\Box_{Chris}\Diamond q$), and at the same time that I can get away with forbidding Kim from eating either ($\neg\Box_{Speaker}\Diamond(p \vee q)$). (12b) is similar: it can mean that Chris needs to allow Kim to eat salad and to allow Kim to eat soup (again $\Box_{Chris}\Diamond p$, $\Box_{Chris}\Diamond q$), but that other people need not allow Kim to eat either.

We can make the same point with (13) and (14), where *believe* and *want* replace *need*:

(13) a. Chris believes that Kim is allowed to eat salad or soup. But I don't.

b. Only Chris believes that Kim is allowed to eat salad or soup.

(14) a. Chris wants to allow Kim to eat salad or soup. But Mary doesn't.
b. Only Chris wants to allow Kim to eat salad or soup.

With the neg-raising property of *believe/want*, we get slightly stronger inferences in (13)-(14) than in (12a)-(12b): in (14a), for example, Chris is understood to want to give Kim permission for either type of food, while I am understood to want to deny Kim both options ($\neg want(\Diamond(p\vee q)) \rightsquigarrow want(\neg\Diamond(p\vee q))$). A similar reading is available in (14b), with *only*'s exclusions feeding the neg-raising property of *want*.

The challenge posed by (12)-(14) is more or less the same, but for illustration consider the cases of VP ellipsis in (12a), (13a) and (14a). In these examples the missing VP is clearly anaphoric to (and by assumption semantically-identical to) a VP of the form $\Box\Diamond(p\vee q)$. In (12a), this is the VP [*need to allow Kim to eat salad or soup*], and in (13a), it is the VP [*believe that Kim is allowed to eat salad or soup*]. Because the negated elided VPs permit the Boolean (unexhaustified) reading noted above ($\neg\Box\Diamond(p\vee q)$), both they and their antecedents must be free of embedded occurrences of Exh. But if this is the case, that is, if the antecedent VPs do not have the LF in (11), where does their apparently embedded FC inference come from?

One conceivable answer is that Exh is in fact embedded in all of the VPs in (12)-(14), including the negated elided VPs, but that its contribution under negation is indistinguishable from its absence. A definition of Exh that has this property, as described in (15), would capture these findings.

(15) **Inertness under negation ($\neg$(~~EXH~~))**:
$\neg\Box(\text{Exh}\Diamond(p\vee q)) \vDash \neg\Box(\Diamond(p\vee q))$

In Sect. 5, I show how (15) can be approached with presuppositional exhaustification (Pex).

The other conceivable answer is that Exh does not appear inside any of VPs in (12)-(14) but that it generates FC in the relevant ones from a position outside of the scope the embedding operator. This requires a definition of Exh that meets (16):

(16) **Global Necessary Free Choice (GLOBAL $\Box$FC)**:
$\text{Exh}(\Box\Diamond(p\vee q)) \vDash \Box\Diamond p \;\&\; \Box\Diamond q$, (i.e., $\text{Exh}(\Box\Diamond(p\vee q)) \vDash \Box(\Diamond p \;\&\; \Diamond q)$)

In Sect. 6, I show how this works on Bar-Lev and Fox's proposal, and the consequence of the proposal to Distribution.

2.2.2 Dataset 2b: free choice under $\Diamond/\exists$

We find similar cases of "embedded" FC when constructions of the form $\Diamond(p\vee q)$ appear in the scope of possibility modals. (17), for instance, can be understood to mean that John's mother is okay with allowing him to have a choice between chocolate and ice cream, and that his father isn't okay with allowing him *either*.

(17) John's mother is okay with allowing him to eat chocolate or ice cream, but his father isn't.

Like in the cases from the previous section, the antecedent and the elided VPs—here of the form $\Diamond\Diamond(p\vee q)$—can only be semantically identical if they are alike in having (or in not having) an embedded occurrence of Exh. If they do, then they would both take their FC interpretation, and if they do not, they would both take their Boolean (non-FC) interpretation. Neither of these is the target reading of (17).

Bar-Lev (2018) makes a similar observation. He notes that FC inferences can be embedded under existential quantifiers. (18) is an example.[4]

(18) a. Some girls are allowed to eat ice cream or cake on their birthday, but no boy is.

b. At least one (of the) girl(s) is allowed to eat ice cream or cake on her birthday, but (it seems that) no boy is.

(18a,b) can mean that there are girls who are permitted to eat ice cream or cake, and have free choice, while no boy is allowed either. Once again we see a case of VP ellipsis where the antecedent VP licenses FC but where the (negated) elided VP takes its Boolean interpretation.[5] How can the two VPs be semantically identical?

The same two possibilities described above present themselves here: either the VPs in (17)-(18) uniformly embed Exh, and under negation the contribution of Exh is effectively vacuous, or the VPs are uniformly vacant of Exh, and the detected FC readings come from exhaustification above the embedding existential operators. These two possibilities require definitions of Exh that meet (19) and (20), respectively.

(19) **Inertness under negation ($\neg$(~~EXH~~))**: (≈ (15))
$\neg\Diamond(\text{Exh}\Diamond(p\vee q)) \vDash \neg\Diamond(\Diamond(p\vee q))$

(20) **Global Existential/Possible Free Choice (GLOBAL $\exists/\Diamond$FC)**:
$\text{Exh}(\Diamond\Diamond(p\vee q)) \vDash \Diamond(\Diamond p \;\&\; \Diamond q)$

(19) and (15) are similar, and I talk about them together in light of presuppositional exhaustification in Sect. 5. I do not know of any definition of Exh that satisfies (20).

3 Background: exclusion and inclusion in exhaustification

In this section, I review two definitions of Exh: one that uses exclusion only (hereafter Exh^{E}, as proposed in Fox 2007), and one that uses exclusion and inclusion ($\text{Exh}^{\text{E+I}}$, as proposed in Bar-Lev and Fox 2020). Both operators conjoin their propositional argument—(the denotation of) their prejacent—with inferences that are determined from a set of propositions C. The elements of C are the (denotations of) the prejacent's formal alternatives.

[4] See Bar-Lev (2018, §1.5, specifically fn. 55).

[5] The sloppy interpretation of the pronouns in (18) is significant. Crnič (2015), Bar-Lev (2018), and Bar-Lev and Fox (2020) take it that the parallelism domains of the two parts of (18) can only be semantically identical (modulo focus marking) if they contain the (focused) subject quantifiers. If this is so, then semantic identity can only be met if either Exh appears in both parallelism domains (between the quantifier and the possibility modal), or appears in neither.

3.1 Exhaustification with exclusion

Given a proposition ϕ and set of (alternative) propositions C, Exh^{E}_C conjoins ϕ with its 'exclusions' given C, hereafter $\boldsymbol{exclusions}_C(\phi)$:

(21) Given proposition ϕ and set of propositions C,
$\text{Exh}^{\text{E}}_C(\phi) = \phi \;\&\; \boldsymbol{exclusions}_C(\phi)$

$\boldsymbol{exclusions}_C(\phi)$ is the conjunction of the negations of those elements of C that are innocently excludable (IE) given ϕ. We say that a proposition ψ (from C) is IE given ϕ, C iff it appears in every subset B of C that satisfies these two conditions:

(22) (i) The negations of B's elements are jointly consistent with ϕ, $(\bigwedge B^{\neg} \;\&\; \phi \nvDash \bot)$
(ii) No proper superset of B that is also a subset of C satisfies (i).

In the case of unembedded disjunction ($\phi = (p \vee q)$, $C = \{p, q, p \wedge q\}$), only the conjunctive alternative $(p \wedge q)$ is IE, because it and only it appears in the subsets of C that satisfy (22i-ii). These are $B_1 = \{p, (p \wedge q)\}$ and $B_2 = \{q, (p \wedge q)\}$: the negations of the elements of B_1 are consistent with $(p \vee q)$, but this is not the case for any proper superset of B_1 from C; likewise for B_2. Therefore, in this case, $(p \wedge q)$ is the only IE alternative, and $\boldsymbol{exclusions}_C(\phi) = \neg(p \wedge q)$.

We obtain a similar result when disjunction is embedded under a possibility expression ($\phi = \Diamond(p \vee q)$, $C = \{\Diamond p, \Diamond q, \Diamond(p \wedge q)\}$). Here too, as the reader may verify, two subsets of C satisfy (22i-ii): $\{\Diamond p, \Diamond(p \wedge q)\}$ and $\{\Diamond q, \Diamond(p \wedge q)\}$. It follows that only $\Diamond(p \wedge q)$ is IE, and $\boldsymbol{exclusions}_C(\phi) = \neg\Diamond(p \wedge q)$.

As a final example, take the case of $\Box(p \vee q)$. Assuming that $C = \{\Box p, \Box q, \Box(p \wedge q)\}$, we predict all the elements of C to be IE, because C itself is the only subset of C that satisfies the conditions (22i-ii): its elements can all be negated consistently with the proposition $\Box(p \vee q)$, and (trivially) there are no proper supersets of C that are subsets of C that also satisfy this criterion. Therefore, when $\phi = \Box(p \vee q)$ and $C = \{\Box p, \Box q, \Box(p \wedge q)\}$, every element of C is IE, and $\boldsymbol{exclusions}_C(\phi) = \neg\Box p \;\&\; \neg\Box q \;\&\; \neg\Box(p \wedge q)$.

And so, with the assumptions made above about the formal alternatives to $(p \vee q)$, $\Diamond(p \vee q)$, and $\Box(p \vee q)$, we may summarize the results so far as follows. (To keep the notation simple, I remove reference to the relevant sets of formal alternatives.)

(23) a. $\text{Exh}^{\text{E}}(p \vee q) = (p \vee q) \;\&\; \neg(p \wedge q)$
b. $\text{Exh}^{\text{E}}(\Diamond(p \vee q)) = \Diamond(p \vee q) \;\&\; \neg\Diamond(p \wedge q)$
c. $\text{Exh}^{\text{E}}(\Box(p \vee q)) = \Box(p \vee q) \;\&\; \neg\Box p \;\&\; \neg\Box q$

Two things are noteworthy here: (i) applying Exh^{E} to $\Diamond(p \vee q)$ does not produce FC, so we have yet to fulfill Desidertaum 2 (FC); (ii) applying Exh^{E} to $\Box(p \vee q)$ produces the distribution, in line with DIST.

3.2 FC from recursive application of Exh^{E}

Building on insights from Kratzer and Shimoyama (2002), Fox (2007) derives FC from recursive exhaustification, that is, from LFs of the form $\text{Exh}^{\text{E}}(\text{Exh}^{\text{E}}\Diamond(p \vee q))$.

Very briefly, the derivation works as follows. Let $B = \{\Diamond p, \Diamond q, \Diamond(p \wedge q)\}$, let $C = \{\mathrm{Exh}^{\mathrm{E}}_B(\Diamond p), \mathrm{Exh}^{\mathrm{E}}_B(\Diamond q), \mathrm{Exh}^{\mathrm{E}}_B(\Diamond(p \wedge q))\}$, and let $\phi = \mathrm{Exh}^{\mathrm{E}}_C(\Diamond(p \vee q))$. Then, as the reader may verify, every element of C is IE given ϕ, and therefore every element of C participates in the strengthening introduced by the upper occurrence of $\mathrm{Exh}^{\mathrm{E}}$. The result, as summarized below, entails FC:

(24) Given $B = \{\Diamond p, \Diamond q, \Diamond(p \wedge q)\}$,
$C = \{\mathrm{Exh}^{\mathrm{E}}_B \Diamond p, \mathrm{Exh}^{\mathrm{E}}_B \Diamond q, \mathrm{Exh}^{\mathrm{E}}_B \Diamond(p \wedge q)\}$,
$\phi = \mathrm{Exh}^{\mathrm{E}}_B \Diamond(p \vee q)$

$$\begin{aligned}
\mathrm{Exh}^{\mathrm{E}}_C(\phi) &= \mathrm{Exh}^{\mathrm{E}}_B \Diamond(p \vee q)\ \& && (=\phi)\\
&\quad \neg\mathrm{Exh}^{\mathrm{E}}_B \Diamond p\ \& \\
&\quad \neg\mathrm{Exh}^{\mathrm{E}}_B \Diamond q\ \& \\
&\quad \neg\mathrm{Exh}^{\mathrm{E}}_B \Diamond(p \wedge q) \\
&= \Diamond(p \vee q)\ \&\ \neg\Diamond(p \wedge q)\ \& && \text{(see (23b))}\\
&\quad \neg(\Diamond p\ \&\ \neg\Diamond q)\ \& \\
&\quad \neg(\Diamond q\ \&\ \neg\Diamond p)\ \& \\
&\quad \neg\Diamond(p \wedge q) \\
&= \Diamond(p \vee q)\ \&\ \neg\Diamond(p \wedge q)\ \& \\
&\quad (\Diamond p \leftrightarrow \Diamond q) \\
&= \Diamond p\ \&\ \Diamond q\ \&\ \neg\Diamond(p \wedge q)
\end{aligned}$$

I leave it also to the reader to verify that recursive exhaustification of unembedded disjunction $(p \vee q)$ and of $\Box(p \vee q)$ does not change the results obtained earlier. This is indicated in the right margins of the updated summary in (25). (Once again, reference to the different sets of formal alternatives is omitted for readability. The assumption throughout these derivations is that the alternatives to any disjunctive prejacent are those that result from replacing the disjunction in the prejacent with its disjuncts and with their conjunction.)

(25) a. $\mathrm{Exh}^{\mathrm{E}}(p \vee q) = (p \vee q)\ \&\ \neg(p \wedge q)$ $\quad (= \mathrm{Exh}^{\mathrm{E}}(\mathrm{Exh}^{\mathrm{E}}(p \vee q)))$

b. $\mathrm{Exh}^{\mathrm{E}}(\mathrm{Exh}^{\mathrm{E}}(\Diamond(p \vee q))) = \Diamond p\ \&\ \Diamond q\ \&\ \neg\Diamond(p \wedge q)$

c. $\mathrm{Exh}^{\mathrm{E}}(\Box(p \vee q)) = \Box(p \vee q)\ \&\ \neg\Box p\ \&\ \neg\Box q$ $\quad (= \mathrm{Exh}^{\mathrm{E}}(\mathrm{Exh}^{\mathrm{E}}(\Box(p \vee q))))$

3.3 Exhaustification with exclusion and inclusion

Bar-Lev and Fox (2020) propose a different account of exhaustification, and with it a different account of FC. The key notion in their proposal is *inclusion*. Given a proposition ϕ and set of (alternative) propositions C, $\mathrm{Exh}^{\mathrm{E+I}}_C$ conjoins ϕ with two propositions: what I called ***exclusions***$_C(\phi)$ earlier, and what I here call ***inclusions***$_C(\phi)$.

(26) Given proposition ϕ and set of propositions C,
$\mathrm{Exh}^{\mathrm{E+I}}_C(\phi) = \phi\ \&\ \textit{\textbf{exclusions}}_C(\phi)\ \&\ \textit{\textbf{inclusions}}_C(\phi)$

inclusions$_C(\phi)$ is the conjunction of those elements of C that are innocently includable (II) given ϕ. A proposition ψ (from C) is II given ϕ, C iff it appears in every subset B of C that satisfies these two conditions:

(27) (i) B's elements are jointly consistent with ϕ & $\boldsymbol{exclusions}_C(\phi)$, $(\bigwedge B \text{ \& } \phi \text{ \& } \boldsymbol{exclusions}_C(\phi) \wedge \not\vDash \bot)$

(ii) No proper superset of B that is also a subset of C satisfies (i).

When $\phi = (p \vee q)$ and $C = \{p, q, (p \wedge q)\}$, the alternative $(p \wedge q)$ is predicted to be IE (as explained earlier), and nothing from C is predicted to be II. This is because ϕ & $\boldsymbol{exclusions}_C(p \vee q)$ in this case is the proposition $(p \vee q)$ & $\neg(p \wedge q)$, and there are two subsets of C that satisfy (27i-ii): $B_1 = \{p\}$ and $B_2 = \{q\}$. B_1's element—there is only one, p—is consistent with $(p \vee q)$ & $\neg(p \wedge q)$, but the same is not true of any proper superset of B_1 from C; likewise for B_2. Therefore there are no II alternatives from C in this case, and $\boldsymbol{inclusions}_C(\phi) = \top$.

With $\phi = \Box(p \vee q)$ and $C = \{\Box p, \Box q, \Box(p \wedge q)\}$, the set of includable alternatives is also empty. In this case the reason is that $\boldsymbol{exclusions}_C(\phi)$ negates every element of C, leaving nothing that can be asserted consistently with ϕ & $\boldsymbol{exclusions}_C(\phi)$.

With $\phi = \Diamond(p \vee q)$ and $C = \{\Diamond p, \Diamond q, \Diamond(p \wedge q)\}$, the alternatives $\Diamond p$, $\Diamond q$, are together consistent with ϕ and its exclusion (i.e., the proposition $\Diamond(p \vee q)$ & $\neg\Diamond(p \wedge q)$). The alternatives $\Diamond p$, $\Diamond q$ are therefore II, so $\boldsymbol{inclusions}_C(\phi) = (\Diamond p \text{ \& } \Diamond q)$. FC therefore results from single application of $\text{Exh}^{\text{E+I}}$ to $\Diamond(p \vee q)$.[6] These results are summarized below:

(28) a. $\text{Exh}^{\text{E+I}}(p \vee q) = (p \vee q) \text{ \& } \neg(p \wedge q)$

b. $\text{Exh}^{\text{E+I}}(\Diamond(p \vee q)) = \Diamond(p \vee q) \text{ \& } \neg\Diamond(p \wedge q) \text{ \& } \Diamond p \text{ \& } \Diamond q$

c. $\text{Exh}^{\text{E+I}}(\Box(p \vee q)) = \Box(p \vee q) \text{ \& } \neg\Box p \text{ \& } \neg\Box q$

4 Exh^{E}, $\text{Exh}^{\text{E+I}}$, and the data

Recall our desiderata. Exhaustification, which by hypothesis is the source of all the inferences discussed above, must satisfy either DIST, FC, and ¬(~~EXH~~), or DIST, FC, GLOBAL □FC, and GLOBAL ◇FC.

(7) **Distribution (DIST)**:
$\text{Exh}(\Box(p \vee q)) \vDash \neg\Box p \text{ \& } \neg\Box q$

(9) **Free choice (FC)**:
$\text{Exh}(\Diamond(p \vee q)) \vDash \Diamond p \text{ \& } \Diamond q$

(15) **Inertness under negation (¬(~~EXH~~))**: (collapsed here with (19))
$\neg\Box(\text{Exh}\Diamond(p \vee q)) \vDash \neg\Box(\Diamond(p \vee q))$
$\neg\Diamond(\text{Exh}\Diamond(p \vee q)) \vDash \neg\Diamond(\Diamond(p \vee q))$

(16) **Global Necessary free choice (GLOBAL □FC)**:
$\text{Exh}(\Box\Diamond(p \vee q)) \vDash \Box\Diamond p \text{ \& } \Box\Diamond q$, (i.e., $\text{Exh}(\Box\Diamond(p \vee q)) \vDash \Box(\Diamond p \text{ \& } \Diamond q)$)

(20) **Global Possible free choice (GLOBAL ◇FC)**:
$\text{Exh}(\Diamond\Diamond(p \vee q)) \vDash \Diamond(\Diamond p \text{ \& } \Diamond q)$

[6] If we assume that any prejacent ϕ is an alternative of itself ($\phi \in C$), then ϕ will always be includable. So strictly speaking there is no need to write ϕ separately in the definition of $\text{Exh}^{\text{E+I}}$ in (26), but I leave it there to keep things clear.

From the demonstrations in the previous section, it is clear that both Exh^{E} and $\text{Exh}^{\text{E+I}}$ satisfy DIST, as long as the assumptions about the formal alternatives are maintained (more on this below). It is also clear that both Exh^{E} and $\text{Exh}^{\text{E+I}}$ satisfy FC, though recursive application is necessary in the case of Exh^{E}.

As things stand, however, neither operator satisfies the remaining desiderata. Because Exh^{E} and $\text{Exh}^{\text{E+I}}$ conjoin their prejacents with their exclusions/inclusions, the negation of Exh^{E}/$\text{Exh}^{\text{E+I}}$ is the negation of those conjunctions, which is too weak to satisfy $\neg$(~~EXH~~). In the next section, we see how this changes with presuppositional exhaustification.

Exh^{E} and $\text{Exh}^{\text{E+I}}$ also fail GLOBAL $\Box$FC and GLOBAL $\Diamond$FC. If the set of formal alternatives to $\Box\Diamond(p\vee q)$ is $C = \{\Box\Diamond p, \Box\Diamond q, \Box\Diamond(p\wedge q)\}$, then every alternative is predicted to be innocently excludable. It follows that (recursive) application of Exh^{E}, and application of $\text{Exh}^{\text{E+I}}$, both conjoin $\Box\Diamond(p\vee q)$ with the negations of its disjunct alternatives $\neg\Box\Diamond p$, $\neg\Box\Diamond q$, not with $\Box\Diamond p$, $\Box\Diamond q$ as desired. This changes when our assumptions about C change, as the discussion in Sect. 6 shows. I argue, however, that making these changes is problematic.

In the case of $\Diamond$FC there does not seem to be a straightforward way of getting the facts right with Exh^{E} or $\text{Exh}^{\text{E+I}}$. When I return to this, I show that (recursive) application of either operator to $\Diamond\Diamond(p\vee q)$ produces only the inference $(\Diamond\Diamond p \;\&\; \Diamond\Diamond q)$, not $\Diamond(\Diamond p \;\&\; \Diamond q)$.

5 Presuppositional exhaustification

5.1 Background

Del Pinal et al. (2022) propose a mode of exhaustification that presupposes the exclusions/inclusions of its prejacent, and asserts the prejacent itself. Let us tentatively define the operator Pex as follows (subscripts indicate presuppositions):

(29) **<u>Pex (first version—to be revised)</u>**:
$\text{Pex}_C^{\text{E+I}}(\phi) = \phi_{\textbf{\textit{exclusions}}_C(\phi)\,\&\,\textbf{\textit{inclusions}}_C(\phi)}$

A question that (29) brings up is whether the presupposition of Pex is of the kind that requires support from common ground prior to utterance, or whether it is of the kind that simply feeds non-at-issue semantic content. I assume the latter, following Del Pinal et al. (2022). Specifically, I assume that Pex's presuppositions are globally accommodated by default, and that when it appears in embedded positions, Pex's presuppositions project like other presuppositions do.[7]

With this aside, let us look at how Pex is predicted to interact with negation, specifically in configurations where it is outscoped by it. This is relevant for our data, and soon it will lead us to a revision of (29).

[7]Del Pinal et al. add the qualification that Pex's presuppositions are accommodated if they are consistent with the common ground. This detail is not relevant for my purposes, so I ignore it here.

Consider first a negated disjunction of the form $\neg\text{Pex}(p \vee q)$.[8] Assuming the alternatives $C = \{p, q, p \wedge q\}$ and the definition in (29), $\neg\text{Pex}_C(p \vee q)$ is simply the negation of $\text{Pex}_C(p \vee q)$, i.e., the negation of $(p \vee q)_{\neg(p \wedge q)}$. If we take it that negation is a presupposition hole, then $\neg\text{Pex}_C(p \vee q)$ will inherit the presuppositions of its negatum, so $\neg\text{Pex}_C(p \vee q) = \neg(p \vee q)_{\neg(p \wedge q)}$. Notice that the exclusion of Pex, in this case the proposition $\neg(p \wedge q)$, does not weaken the global assertoric content $\neg(p \vee q)$; it is entailed by it. And on the assumption that Pex's presuppositions do not make requirements on prior conversational background, $\neg\text{Pex}_C(p \vee q)$ turns out to be effectively equivalent to $\neg(p \vee q)$. This feature is relevant given the desideratum that I called $\neg$(~~EXH~~) earlier.

Now consider a negated FC disjunction, of the form $\neg\text{Pex}\Diamond(p \vee q)$. Here the exclusion of Pex is $\neg\Diamond(p \wedge q)$, and its inclusion is FC ($\Diamond p$, $\Diamond q$). Both are predicted to be presupposed under negation, and to project through negation accordingly. The result in this case is a contradiction: the presupposed inclusions $\Diamond p$, $\Diamond q$ are inconsistent with the assertoric content of $\neg\text{Pex}(\Diamond(p \vee q))$, $\neg\Diamond(p \vee q)$. Notice that $\Diamond p$, $\Diamond q$ are includable at the level of Pex in this configuration, since they are very much consistent with Pex's (unnegated) prejacent $\Diamond(p \vee q)$.

At first this outcome may not seem immediately worrying. We might say that embedding Pex under negation is optional, and that the option is dropped, as it were, whenever it leads to contradictory results. In other words, a sentence of the form $\neg\Diamond(p \vee q)$ may be parsed either with Pex under negation or without it, and because the former is contradictory, it is simply not considered as a possible parse of the sentence. From this, we predict that sentences of the form $\neg\Diamond(p \vee q)$ only have their Boolean interpretation, and this is indeed the case, as Alonso-Ovalle (2005) and many since have pointed out.

However, there are other FC configurations in which the presence of Pex can be motivated independently, and that are consequently predicted to produce this contradiction, contrary to fact. These call for a revision of (29).

Consider (30). Intuitively the sentence has a reading where *only*'s presupposed prejacent is understood to license FC, but where *only*'s asserted exclusions are understood to negate the Boolean interpretations of the focus alternatives:[9]

(30) Only Kim is allowed to eat salad or soup
Presupposes: Kim is allowed to eat salad, allowed to eat soup $\Diamond p$, $\Diamond q$
Asserts: Others are not allowed to eat either $\neg\Diamond p$, $\neg\Diamond q$

I assume that *only* takes one propositional argument, its prejacent, and presupposes its truth for the given focus associate (here *Kim*) and asserts its falsity for the focus alternatives (alternatives where *Kim* is replaced with other relevant names). If the prejacent is parsed with Pex, then the presupposition should license FC, but in this case the negations of the focus-alternatives should (each) be contradictory. These are shown in (31b); in each of them, Pex asserts $\neg\Diamond(p \vee q)$ but presupposes $\Diamond p$, $\Diamond q$.

[8] Applying Pex above negation is of course possible, but it does not produce a significantly different result from applying Exh above negation. The exclusions/inclusions in these cases, if there are any, are predicted to be presupposed and (by hypothesis) freely accommodated, which is the same as the result we get from applying Exh. I therefore do not discuss these parses in this section.

[9] Alxatib (2020).

(31) Only Kim is Pex(allowed to eat salad or soup)

a. *Presupposition*: Kim is Pex(allowed to eat salad or soup)
$\Longrightarrow$ Kim is allowed to eat salad;
$\Longrightarrow$ Kim is allowed to eat soup

b. *Assertion*: $\neg$(Chris is Pex(allowed to eat salad or soup)), ($\perp$)
$\Longrightarrow$ $\neg$(Chris is allowed to eat salad or soup)
$\Longrightarrow$ Chris is allowed to eat salad
$\Longrightarrow$ Chris is allowed to eat soup

$\neg$(Jessie is Pex(allowed to eat salad or soup)), ($\perp$)
$\Longrightarrow$ $\neg$(Jessie is allowed to eat salad or soup)
$\Longrightarrow$ Jessie is allowed to eat salad
$\Longrightarrow$ Jessie is allowed to eat soup

On the other hand, if *only*'s prejacent in (30) is parsed without Pex, the negations of the focus-alternatives are not contradictory—see (32b) below—but the presupposed prejacent is predicted to *not* license FC—(32a). The resulting reading in this case is consistent and available, but it is not the target reading.[10]

(32) Only Kim is allowed to eat salad or soup

a. *Presupposition*: Kim is allowed to eat salad or soup
$\not\Longrightarrow$ Kim is allowed to eat salad;
$\not\Longrightarrow$ Kim is allowed to eat soup

b. *Assertion*: $\neg$ (Chris is allowed to eat salad or soup), ($\checkmark$)
$\neg$(Jessie is allowed to eat salad or soup), ($\checkmark$)
...

To get the target reading, we need to define Pex in a way that escapes the contradiction it produces under negation, but at the same time produces FC in positive contexts. To this end, and following similar but different empirical arguments as the one above, Del Pinal et al. (2022) propose the revision in (33). On this definition Pex does not assert its prejacent's includable alternatives, but imposes the weaker requirement that they be homogeneous, that is, that they have the same truth value.[11] (The set of includable alternatives is indicated below as $\mathrm{II}_C(\phi)$, and H is the function whose output, given a set of propositions as input, is the proposition that the elements of the input have the same truth value.)

(33) **Pex (revised)**:
$\mathrm{Pex}_C^{\mathrm{E+H}}(\phi) = \phi_{\boldsymbol{exclusions}_C(\phi)\,\&\,H(\mathrm{II}_C(\phi))}$
Where for any set of propositions S,
$$H(S) = [\lambda w\,.\,\forall\phi\forall\psi(\phi, \psi \in S \rightarrow \phi(w) = \psi(w))]$$

[10] There remains the parse in which Pex appears above *only*. The resulting inferences in this case are consistent with one another and with the facts, but as the reader may verify, they make *only* vacuous and therefore infelicitous.

[11] See related suggestions in Simons (2001) and Chemla (2009a).

With this definition of Pex, our earlier result for FC configurations changes slightly: FC is not directly presupposed by Pex, but comes from the homogeneity presupposition $\Diamond p \leftrightarrow \Diamond q$ and Pex's assertion of its prejacent $\Diamond(p \vee q)$:[12]

(34) When $\phi = \Diamond(p \vee q)$ and $C = \{\Diamond p, \Diamond q, \Diamond(p \wedge q)\}$,

a. $\boldsymbol{exclusions}_C(\phi) = \neg\Diamond(p \wedge q)$.
$H(\mathrm{II}_C(\phi)) = \Diamond p \leftrightarrow \Diamond q$.

b. $\mathrm{Pex}_C^{\mathrm{E+H}}(\phi) = \Diamond(p \vee q)_{\neg\Diamond(p\wedge q)\ \&\ (\Diamond p \leftrightarrow \Diamond q)}$

c. ... (See below)

The relevant consequence to our discussion is that Pex can now be interpreted under negation in FC configurations without leading to a contradiction. Pex's homogeneity inference $\Diamond p \leftrightarrow \Diamond q$, like its exclusive inference $\neg\Diamond(p \wedge q)$, follows from $\neg\Diamond(p \vee q)$. Therefore we predict once again the absence of FC in sentences of the form $\neg\Diamond(p \vee q)$.

(34) c. $\neg\mathrm{Pex}(C)(\phi) = \neg\Diamond(p \vee q)_{\neg\Diamond(p\wedge q)\ \&\ (\Diamond p \leftrightarrow \Diamond q)}$

We now derive FC in the presupposition of *only* in (30), but not in its assertion. The result is summarized below.

(35) Only Kim is Pex(allowed to eat salad or soup)

a. *Presupposition*: Kim is Pex(allowed to eat salad or soup)
$\Longrightarrow$ Kim is allowed to eat salad;
$\Longrightarrow$ Kim is allowed to eat soup (given (34b))

b. *Assertion*: $\neg$(Chris is Pex(allowed to eat salad or soup)),
$\Longrightarrow$ $\neg$(Chris is allowed to eat salad or soup)
$\Longrightarrow$ Chris is not allowed to eat salad
$\Longrightarrow$ Chris is not allowed to eat soup (given (34c))

$\neg$(Jessie is Pex(allowed to eat salad or soup)),
$\Longrightarrow$ $\neg$(Jessie is allowed to eat salad or soup)
$\Longrightarrow$ Jessie is not allowed to eat salad
$\Longrightarrow$ Jessie is not allowed to eat soup (given (34c))

5.2 Back to the data

We may now return to our desiderata, specifically $\neg$(~~EXH~~), and the ellipsis contexts that they were based on.

5.2.1 Revisiting Dataset 2a: free choice under $\Box$

Consider (36) again, repeated from (13a).

(36) Chris believes that Kim is allowed to eat salad or soup. But I don't.

[12] This has the desirable consequence that FC, unlike other implicatures, is predicted to be an at-issue inference. I suspect that this result can help explain why FC cannot be canceled with *in fact* continuations (Barker 2010), but I leave the details to another occasion.

Consider now the following two parses of (36):

(37) a. Chris Pex [believes that [Kim is allowed to eat salad or soup]], but I don't [believe that [Kim is allowed to eat salad or soup]].

b. Chris [believes that [Pex [Kim is allowed to eat salad or soup]]], but I don't [believe that [Pex [Kim is allowed to eat salad or soup]]].

In (37a) Pex appears outside the antecedent VP [*believe that...*], and it does not accompany the negated elided VP. In (37b) Pex appears embedded under *believe* in both the antecedent and the elided VPs.

On both of (37a,b) semantic identity holds between the antecedent and the elided VPs. (37a) produces the Boolean reading of the elided VP, as desired, but because Pex appears above the antecedent VP—above *believe*, that is—we predict exhaustification to produce the negations of the disjunct alternatives; as noted earlier, $\Box\Diamond p$ and $\Box\Diamond q$ are innocently excludable, given $\Box\Diamond(p\vee q)$ and the set of formal alternatives $C = \{\Box\Diamond p, \Box\Diamond q, \Box\Diamond(p\wedge q)\}$. This is the opposite of the target embedded FC inference.

In (37b) the predictions are more complicated. Because Pex by assumption presupposes the inferences that it adds to its prejacent, its effect on the overall meaning of (37b) depends on how its presuppositions project through the context that embeds it, in this case the verb *believe*.

There is consensus in contemporary literature that *believe* projects presuppositions of belief (ever since Karttunen 1973, 1974). This is based on the finding that (38a,b) do not intuitively presuppose anything; once it is stated that Bill believes that I have a bike/had falafel for lunch, subsequent discourse can continue where Bill's beliefs are assumed to support the propositions that I have a bike/had falafel for lunch.

(38) a. Bill believes that I have a bike, and he believes that my bike is expensive.

b. Bill believes that I had falafel for lunch, and he believes that I will have falafel again for dinner.

Accepting, then, that for any presuppositional sentence S_ϕ, [A *believes* S_ϕ] presupposes that A believes ϕ, we predict that [A *believes* Pex(S)] presuppose that A believes the presuppositions of Pex, i.e., the exclusions/homogeneity inferences of S. So (37b) should presuppose, or at least imply, (i) that both Chris and I believe that Kim is not permitted to eat salad and soup (*believe*$(\neg\Diamond(p\wedge q))$), and (ii) that both of us believe homogeneity (*believe*$(\Diamond p \leftrightarrow \Diamond q)$).

The assertions of the two parts of (37b) differ of course. The first part says that Chris believes $\Diamond(p\vee q)$, and given the presuppositions in (i,ii) above, it follows that Chris believes that Kim has free choice. The second part says that I believe $\neg\Diamond(p\vee q)$—this follows from the negation of *believe* together with the verb's neg-raising property—and since this entails both of (i,ii), the presence of Pex is predicted to be effectively vacuous (recall $\neg(\cancel{\text{EXH}})$). So in sum, the LF in (37b) produces the target reading of (36).

Our other examples that instantiate $\Box\Diamond(p\vee q)$ are less straightforward. Consider the desire report in (39), repeated from (14a).

(39) Chris wants to allow Kim to eat salad or soup. But I don't.

The parses of interest in this case are in (40), and as in the case of *believe*, interpreting Pex above *want* (as in (40a)) will not give us the target embedded FC reading. We therefore consider only (40b).

(40) a. Chris Pex [wants to [allow Kim to eat salad or soup]],
but I don't [want to [allow Kim to eat salad or soup]].

b. Chris [want to Pex [allow Kim to eat salad or soup]],
but I don't [want to Pex [allow Kim to eat salad or soup]].

How does *want* project the presuppositions of its complement? The answer is not as clear as it is for *believe*. From the presuppositionless discourses in (41a,b) we may conclude that [A *wants* S_ϕ] presupposes that A believes ϕ—more accurately that its presupposition is entailed by the proposition that A believes ϕ.

(41) a. Bill believes that has a bike, and he wants to buy a rack for his bike.

b. Bill believes that he had falafel for lunch, and he wants to have falafel again for dinner.

However, (42a,b) point to a different conclusion: that the presupposition of [A *wants* S_ϕ] is (or at least entailed by) the proposition that A wants ϕ.

(42) a. Bill wants to get a bike, and he wants to buy a rack for his bike.

b. Bill wants to have falafel for lunch, and he wants to have falafel again for dinner.

Examples like (41) and (42) have kept open the question whether *want* projects belief presuppositions or desire presuppositions. Some, like Heim (1992), propose that *want* projects a belief presupposition (following Karttunen 1974), and that the apparent filtering in *want*-*want* sequences comes from a mechanism of modal subordination (Roberts 1989). On the other hand, Schlenker's (2008) transparency theory of presupposition projection predicts that *want* project desire rather than belief presuppositions. The significance of this prediction has been recently discussed by Blumberg and Goldstein (2022), who argue that it is in principle possible to adopt Schlenker's predictions, and to account for the filtering facts in (41a,b), rather than (42a,b), using modal subordination (see Blumberg and Goldstein 2022, §6).

If we accept the predictions of Schlenker's proposal, and assume that *want* projects desire presuppositions, then (40b) receives essentially the same analysis as (37b). The two parts of (40b) should presuppose (i) that neither Chris nor I want to give Kim permission to eat both salad and soup ($want(\neg\Diamond(p\wedge q))$), and (ii) that both of us want homogeneity ($want(\Diamond p \leftrightarrow \Diamond q)$). These, respectively, are the desire projections of Pex's presuppositions ($\neg\Diamond(p\wedge q)$ and $(\Diamond p \leftrightarrow \Diamond q)$). The inferences $want(\neg\Diamond(p\wedge q))$ and $want(\Diamond p \leftrightarrow \Diamond q)$, together with the assertion of the first part of (40b), $want(\Diamond(p\vee q))$, produce the embedded FC reading (Chris's desire worlds are FC worlds), and they do not add anything to the assertion of the second half ($want(\neg\Diamond(p\vee q))$). The occurrence of Pex there is thus made inert here too.

On the other hand, if we take it that *want* projects belief presuppositions instead (following Karttunen and Heim), then the predictions are problematic: from (39), given LF (40b), we should get the inference that Chris (and the speaker) believe that

Kim is not permitted to each both soup and salad, and believe that the two food items are either each permitted or each not permitted (homogeneity). I do not think that (39) licenses these inferences intuitively, so at the moment this remains a potential challenge to the account.

With other instantiations of $\Box$, we encounter additional questions. Recall the example with *need*, repeated as (43) below. Look also at the LF of interest in (44)

(43) Chris needs to allow Kim to eat salad or soup, but I don't. (=12)

(44) Chris [needs to Pex [allow Kim to eat salad or soup]],
but I don't [need to Pex [allow Kim to eat salad or soup]].

When *need* embeds a presupposition trigger, it behaves (on first glance) as a presupposition hole, but on second glance we find (in parallel to the cases of *believe* and *want*) that the presuppositions are filtered by earlier necessity statements. (45a,b) are examples.

(45) a. Bill needs to get a bike, and he needs to buy a rack for his bike.
b. Bill needs to eat falafel for lunch, and he needs to eat falafel again for dinner.

Indeed, going by Schlenker's theory, the prediction is parallel to that of *want* and *believe*: [*need* S_ϕ] presupposes [*need* ϕ]. From these considerations we make the following prediction: the first part of (44) should presuppose that Pex's presuppositions (its exclusions/inclusions) are necessary, i.e., that it is necessary for Chris to prohibit Kim from eating both salad and soup ($need(\neg\Diamond(p\wedge q))$), and that it is necessary to permit the individual items consistently ($need(\Diamond p \leftrightarrow \Diamond q)$). With the assertion $need(\Diamond(p\vee q))$, the embedded FC inference follows ($need(\Diamond p\ \&\ \Diamond q)$), as desired. In the second half, the presuppositions project through negation, giving us the inferences $need(\neg\Diamond(p\wedge q))$ and $need(\Diamond p \leftrightarrow \Diamond q)$, along with the assertion $\neg need(\Diamond(p\vee q))$. Here I think the prediction is incorrect. (44) should imply that the speaker needs to be consistent in permitting or prohibiting the two kinds of food, but the inference is not available as far as I can tell: (43) can be said truthfully in a scenario where there are no requirements at all on the part of the speaker. This too is a challenge to the account.

The general result so far is that our target readings—intuitively, embedded FC—may be accounted for if we assumed parses of the form $\Box\text{Pex}\Diamond(p\vee q)$ in both the affirmative and the negated sentences in our examples. The details remain to be fully worked out, however, given that the predictions of the account depend crucially on the projection profiles of the different instantiations of $\Box$, which are not yet understood.

5.2.2 Revisiting Dataset 2b: free choice under $\Diamond/\exists$

We turn finally to Dataset 2b, where FC appears to be interpreted in the scope of existential quantifiers and possibility modals ((46) and (47), respectively). The examples are repeated in reverse order from (17)-(18).

(46) a. Some girls are allowed to eat ice cream or cake on their birthday, but no boy is. (= (18a))

b. At least one (of the) girl(s) is allowed to eat ice cream or cake on her birthday, but (it seems that) no boy is. (= (18b))

(47) John's mother is okay with allowing (/to allow) him to eat chocolate or ice cream, but his father isn't. (= (17))

Consider the following LF of (46b):

(48) At least one (of the) girl(s) is Pex allowed to eat ice cream or cake on her birthday,
but (it seems that) no boy is Pex allowed to eat ice cream or cake on his birthday.

The predictions of our account depend on how existential quantifiers project the presuppositions of their scope, a question that has seen extensive discussion in the literature and one that I cannot do justice to here. If we borrow from Heim (1983) the assumption that existential quantifiers project universal presuppositions—i.e., that [*Some NP* VP_{ϕ}] presupposes that every element of the restrictor satisfies ϕ—then we produce existential FC in the first half of (48), as desired.[13] However, the result comes at the price of a universal homogeneity inference, which in this case is the inference that every (relevant) girl is allowed one dessert iff she is allowed the other. The inference is not intuitively available, but this is a familiar problem: intuitively, existential quantifiers are not always felt to license universal presuppositions (e.g., (49)), so this problematic prediction is not specific to the Pex-based account of existential FC.

(49) At least one of our students quit smoking (this year).
$\not\Rightarrow$ All of our students were smokers.

Getting the details right in the case of (48) therefore requires a theory of presupposition projection that gets things right in the case of (49), so at this point, I simply note that the Pex-based account provides a plausible (if yet to be fully articulated) account of existential FC. Crucially, the embedded occurrence of Pex in the second half of (48) is correctly predicted not to enrich the negated content; the quantifier *no boy* projects the universal presupposition that every boy satisfies homogeneity and the exclusive inference ($\forall x(Bx \rightarrow (\Diamond Px \leftrightarrow \Diamond Qx))$), and $\forall x(Bx \rightarrow (\neg\Diamond(Px \wedge Qx)))$), both of which follow from the assertion $\neg\exists x(Bx \,\&\, \Diamond(Px \vee Qx))$.

In the case of possibility statements, like (47), the overall logic of the analysis is parallel to the analysis of (46). We ask first how expressions of possibility project the presuppositions of their complements. As far as I can tell, such expressions behave like holes when they are uttered out of the blue (e.g., (50a)), but the constructions seem to also be subject to filtering in discourses where the presupposition is said to be necessary (50b), desired (50c), or permitted (50d).[14]

(50) a. John's mother is okay with him having salad again for dinner.
Presupposition: John had salad earlier in the day

[13] The assertion $\exists x(\Diamond(Px \vee Qx))$ and the presupposition $\forall x(\Diamond Px \leftrightarrow \Diamond Qx)$ entail $\exists x(\Diamond Px \,\&\, \Diamond Qx)$.

[14] Gazdar (1979) made similar observations about epistemic possibility modals—see his examples (79)-(81), p. 111.

b. It is necessary for John to eat salad for lunch, and it is okay for him to have salad again for dinner.
c. John's mother wants him to eat a salad for lunch, and she is okay with him having salad again for dinner.
d. John's mother is okay with him eating ice cream after lunch, and she is okay with him eating ice cream again after dinner.

There is much more that needs investigation in these data than I can take on here, specifically considering that the effects in (50b-d) may turn out to be due to modal subordination. With respect to the Pex-based account of "possible/permitted" FC (like (47)), the logic is essentially the same as it is for existential quantifiers: the target embedded FC inference—on a non-dynamic theory of presupposition projection—requires that expressions of possibility project a universal presupposition over their domain of (relevant) possible worlds. $\Diamond(\text{Pex}\Diamond(p \vee q))$ would then assert $\Diamond(\Diamond(p \vee q))$ and presuppose $\Box(\Diamond p \leftrightarrow \Diamond q)$, and together the two propositions produce the target inference $\Diamond(\Diamond p \ \& \ \Diamond q)$.[15]

6 Global exhaustification and inclusion

The other path to our embedded FC inferences involves deriving them from exhaustification of $\Box\Diamond(p \vee q)$ and $\Diamond\Diamond(p \vee q)$, respectively. This is possible if exhaustification meets GLOBAL $\Box$FC and GLOBAL $\Diamond$FC, repeated below.

(16) **Global Necessary free choice (GLOBAL $\Box$FC):**
$\text{Exh}(\Box\Diamond(p \vee q)) \vDash \Box\Diamond p \ \& \ \Box\Diamond q$ (i.e., $\text{Exh}(\Box\Diamond(p \vee q)) \vDash \Box(\Diamond p \ \& \ \Diamond q)$)

(20) **Global Possible free choice (GLOBAL $\Diamond$FC):**
$\text{Exh}(\Diamond\Diamond(p \vee q)) \vDash \Diamond(\Diamond p \ \& \ \Diamond q)$

We saw in Sect. 3 that GLOBAL $\Box$FC cannot be achieved if the formal alternatives to $\Box\Diamond(p \vee q)$ are taken to be $\Box\Diamond p$, $\Box\Diamond q$, and $\Box\Diamond(p \wedge q)$; these alternatives are innocently excludable with respect to $\Box\Diamond(p \vee q)$, so applying either Exh^{E} or $\text{Exh}^{\text{E+I}}$ (or $\text{Pex}^{\text{E}}/\text{Pex}^{\text{E+I}}$) to $\Box\Diamond(p \vee q)$ is predicted to produce $\neg\Box\Diamond p$ and $\neg\Box\Diamond q$ as inferences.

Bar-Lev and Fox (2020) discuss a similar challenge that comes from cases like (51).[16]

(51) Every girl was allowed to eat ice cream or cake on her birthday.
But interestingly, no boy was.

The first part of (51)—hereafter $\forall x(\Diamond(Px \vee Qx))$—intuitively licenses a universal FC inference. But as in the case of $\Box\Diamond(p \vee q)$, applying exhaustification above the universal quantifier produces the opposite result: the universal disjunct alternatives $\forall x(\Diamond Px)$, $\forall x(\Diamond Qx)$ are innocently excludable, so (51) should license the inference

[15] Universal projection is predicted by Schlenker (2008, 2009) for both existential quantifiers and possibility expressions.

[16] Based on Crnič (2015) and Chemla (2009b). Recall the relevance of pronoun binding, summarized in fn. 5.

that not every girl was allowed to eat ice cream, and not every girl was allowed to eat cake.

But Bar-Lev and Fox point out that if the formal alternatives to constructions like (51) are assumed to include the existential disjunct alternatives ($\exists x(\Diamond Px)$, $\exists x(\Diamond Qx)$) in addition to the universal ones ($\forall x(\Diamond Px)$, $\forall x(\Diamond Qx)$), exhaustification above the universal quantifier produces the target reading.

To keep things relevant to our discussion, I show how Bar-Lev and Fox's proposal might work with our examples, e.g., (52), where *believe* (an instantiation of $\Box$) replaces the universal quantifier from (51).

(52) Chris believes that Kim is allowed to eat salad or soup. But I don't. (= (36))

The parse of interest now is one where $\text{Exh}^{\text{E+I}}$ appears above *believe*:

(53) Chris $\text{Exh}^{\text{E+I}}$ [believes that [Kim is allowed to eat salad or soup]],
but I don't [believe that [Kim is allowed to eat salad or soup]].

$\text{Exh}^{\text{E+I}}$'s prejacent in (53) is of the form $\Box\Diamond(p\vee q)$. Assume (as Bar-Lev and Fox suggest) that the prejacent's alternatives are $\Box\Diamond p$, $\Box\Diamond q$, $\Diamond\Diamond p$, $\Diamond\Diamond q$, and the conjunctive alternative is $\Box\Diamond(p\wedge q)$.[17] It is worth remarking here that, unlike in the case of the universal quantifier, it isn't obvious what lexical item can instantiate the upper "weak" operator $\Diamond$ in these proposed alternatives—neither *believe* nor *want* have lexical duals in English.[18] This aside, it can now be shown that, with $C=\{\Box\Diamond p, \Box\Diamond q, \Diamond\Diamond q, \Diamond\Diamond q, \Box\Diamond(p\wedge q)\}$, exhaustification of $\Box\Diamond(p\vee q)$ produces the target inferences $\Box\Diamond p$, $\Box\Diamond q$: aside from the conjunctive alternative, none of the elements of C are innocently excludable, and they are all innocently includable.[19] This, then, is the result:

(54) When $\phi=\Box\Diamond(p\vee q)$ and $C=\{\Box\Diamond p, \Box\Diamond q, \Diamond\Diamond p, \Diamond\Diamond q, \Box\Diamond(p\wedge q)\}$,

a. $\textit{\textbf{exclusions}}_C(\phi)=\neg\Box\Diamond(p\wedge q)$

b. $\textit{\textbf{inclusions}}_C(\phi)=\Box\Diamond p \;\&\; \Box\Diamond q$

c. $\text{Exh}_C^{\text{E+I}}(\phi)=\Box\Diamond(p\vee q)\;\&$ $\quad(\phi)$
$\neg\Box\Diamond(p\wedge q)\;\&$ $\quad(\textit{\textbf{exclusions}}_C(\phi))$
$\Box\Diamond p\;\&\;\Box\Diamond q$ $\quad(\textit{\textbf{inclusions}}_C(\phi))$

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[17] The alternative $\Diamond\Diamond(p\wedge q)$ is left out of C here. Keeping it would produce a stronger exclusion: $\neg\Diamond\Diamond(p\wedge q)$.

[18] Notice that, if, e.g., *believe* and *want* had a dual $\Diamond$ as an alternative, we predict that, e.g., $\neg$*believe*(ϕ) has $\neg\Diamond(\phi)$ as an alternative. Depending on what we assume the literal semantics of $\neg$*believe*(ϕ) to be, there is a risk that the alternativehood of $\neg\Diamond(\phi)$ would block the possibility of deriving the strengthened "neg-raising" inference of $\neg$*believe*(ϕ). An altogether different possibility—one that does not require *believe* and *want* to have a dual—is that it is the very property of "neg-raising" that strengthens the exclusion of, e.g., *want*$(\Diamond p)$ and *want*$(\Diamond q)$ to *want*$(\neg\Diamond p)$ and *want*$(\neg\Diamond q)$, and that this keeps the alternatives from being excludable, given *want*$(\Diamond(p\vee q))$. The problem with this possibility is that it does not predict any kind of distribution in the case of *want*$(p\vee q)$.

[19] Explanation: the prejacent $\Box\Diamond(p\vee q)$ can be rewritten equivalently as $\Box(\Diamond p\vee\Diamond q)$. If the alternative $\Diamond\Diamond q$ is false, it follows that $\Box\neg\Diamond q$. From $\Box(\Diamond p\vee\Diamond q)$ and $\Box\neg\Diamond q$, it follows that $\Box\Diamond p$. Therefore the alternative $\Box\Diamond p$ cannot be consistently negated together with $\Diamond\Diamond q$, so neither alternative is innocently excludable. Likewise for $\Box\Diamond q$ and $\Diamond\Diamond p$. These alternatives are all *in*cludable, because they are simultaneously consistent with the predicted exclusion of the conjunctive alternative.

Thus by assuming additional formal alternatives to the prejacent $\Box\Diamond(p\vee q)$—alternatives that result from simultaneously replacing $\Box$ with $\Diamond$ and $(p\vee q)$ with its disjuncts, respectively—we achieve GLOBAL $\Box$FC.

An important consequence of this account—specifically its reliance on adding the noted alternatives—is that it makes it difficult to achieve DIST, repeated below:

(7) **Distribution (DIST)**:
Exh$(\Box(p\vee q)) \vDash \neg\Box p$ & $\neg\Box q$

In making this claim, I am assuming that any theory of alternative generation has to treat the cases of $\Box\Diamond(p\vee q)$ and $\Box(p\vee q)$ uniformly; if $\Box\Diamond(p\vee q)$ has the disjunct alternatives $\Diamond\Diamond p$, $\Diamond\Diamond q$, then $\Box(p\vee q)$ should also have its dual disjunct alternatives $\Diamond p$, $\Diamond q$. But as I explain, the robustness of the distribution inference of $\Box(p\vee q)$ (assuming that it is robust—see below) is reason to reject positing $\Diamond p$, $\Diamond q$ as alternatives to $\Box(p\vee q)$.[20]

Recall first our examples of distribution, repeated from (4)-(6).

(55) Bill needs to take Syntax 1 or Semantics 1.
$\rightsquigarrow$ Bill does not need to take Syntax 1
$\rightsquigarrow$ Bill does not need to take Semantics 1

(56) Bill believes/thinks that I play the piano or the electric organ.
$\rightsquigarrow$ It is not the case that Bill believes that I play the piano
$\rightsquigarrow$ It is not the case that Bill believes that I play the electric organ

(57) My supervisor wants me to read *On Denoting* or *On Referring*.
$\rightsquigarrow$ It is not the case that my supervisor wants me to read *On Denoting*
$\rightsquigarrow$ It is not the case that my supervisor wants me to read *On Referring*

Suppose $\Diamond p$, $\Diamond q$ were alternatives to $\Box(p\vee q)$, alongside $\Box p$, $\Box q$, and $\Box(p\wedge q)$. Then, only the conjunctive $\Box(p\wedge q)$ would be IE; the disjunct alternatives here block one another from being IE, just as their analogs block one another from being IE, given $\Box\Diamond(p\vee q)$.[21] In consequence, $\Diamond p$, $\Diamond q$ are predicted to be innocently includable,[22] and the only inference we derive from $\Box(p\vee q)$ is that the disjuncts are possible/permitted: $\Diamond p$, $\Diamond q$. But this inference is weak; it is compatible with scenarios where one of the disjuncts is necessary (or required, desired, believed, etc.), but intuitively, the examples above are not felicitous in such scenarios. From this, I take it

[20] I do not know if there are good theoretical reasons why $\Diamond p$, $\Diamond q$ should not be alternatives to $\Box(p\vee q)$, but I also do not know if there are good theoretical reasons why they should be. The issue is related to a more general question that has come up in the literature: should alternative generation permit replacement of multiple items in the prejacent? If the answer is no, then there would be reason to not consider $\Diamond p$, $\Diamond q$ as alternatives of $\Box(p\vee q)$. The issue is complicated, however. I refer interested readers to discussions in Magri (2009), Chemla and Spector (2011), Romoli (2012), Trinh and Haida (2015), and Breheny et al. (2018). See also Gotzner and Romoli (2018), Geurts and Pouscoulous (2009), and Ippolito (2010) for discussions of configurations where a modal/intensional verb embeds a scalar item.

[21] Here the conjunctive alternative $\Diamond(p\wedge q)$ has consequences on includability: if $\Diamond(p\wedge q)$ were in C, the alternative would be excludable, but its exclusion would block all other alternatives from being includable; $\Diamond p$ would not be includable because its assertion alongside $\Box q$ contradicts $\neg\Diamond(p\wedge q)$ Therefore $\Diamond p$ does not appear in every subset of C that satisfies (i,ii) from the includability condition. The alternative $\Diamond q$ is not includable for the same reason (given $\Box p$).

[22] $\Box p$, $\Box q$ are not II because they cannot be consistently asserted given the exclusion $\neg\Box(p\wedge q)$.

that $\Box(p \vee q)$—more specifically, our instantiations of it—do not have (instantiations of) $\Diamond p$, $\Diamond q$ as formal alternatives, and therefore that we have no independent support for the idea of adding $\Diamond\Diamond p$, $\Diamond\Diamond q$ to the set of alternatives to $\Box\Diamond(p \vee q)$. This leaves the account of GLOBAL $\Box$FC reviewed above unmotivated.

Before I move on to GLOBAL $\Diamond$FC, I want to take note of Ramotowska et al.'s (2022) recent experimental investigation of DIST. As I said in fn. 3, Ramotowska et al. found that speakers accept sentences of the form $\Box(p \vee q)$ as descriptions of situations where $\Box p$ and $\Diamond q$ are true.[23] For example, in a visual context where three boxes are presented, all containing a yellow ball and one containing an additional blue ball, speakers accept the sentence *The mystery box must contain a blue ball or a yellow ball* when they are told that the mystery box has the same contents as one of the three visible boxes. If the disjunctive sentence $\Box(p \vee q)$ came with the distribution inference $\neg\Box p$, $\neg\Box q$, we expect that speakers reject the sentence.

I do not have enough to say about this finding, unfortunately. Clearly, if we take from it that DIST should not be a goal of our theory of exhaustification, then my criticism of Bar-Lev and Fox's proposal would no longer be valid. But I think it is hasty to interpret the results in this way. First, positing the weaker disjunct alternatives $\Diamond p$, $\Diamond q$ to $\Box(p \vee q)$ is not the only way to capture Ramotowska et al.'s results.[24] Second, speakers robustly take sentences of the form $\Box(p \vee q)$ to imply $\neg\Box p$, $\neg\Box q$—this is acknowledged by Ramotowska et al.—so it is not clear how this can be reconciled with the experimental data. Perhaps something about the context of the experiment provides access to the weaker alternatives $\Diamond p$, $\Diamond q$, and perhaps ordinary use does not. But if this is so, we expect the comparable cases of the form $\Box\Diamond(p \vee q)$ to be interpreted without access to their own weak disjunct alternatives, i.e., $\Diamond\Diamond p$, $\Diamond\Diamond q$, so the relevant examples (e.g., (39), repeated below) should strongly favor readings that entail $\neg\Box\Diamond p$, $\neg\Box\Diamond q$, i.e., the negation of the readings reported above. This is not what we find, however. As I said in fn. 3, I became aware of Ramotowska et al.'s results in the later stages of preparing this paper, so I cannot say more about their relevance to my claims.

(39) Chris wants to allow Kim to eat salad or soup. But I don't.

Finally, Bar-Lev and Fox's account does not meet GLOBAL $\Diamond$FC (recall (17)), nor can it derive FC in examples like (18).

(17) John's mother is okay with allowing him to eat chocolate or ice cream, but his father isn't.

(18) a. Some girls are allowed to eat ice cream or cake on their birthday, but no boy is.
b. At least one (of the) girl(s) is allowed to eat ice cream or cake on her birthday, but (it seems that) no boy is.

As Bar-Lev (2018) notes, applying $\mathrm{Exh}^{\mathrm{E+I}}$ above the existential quantifiers in (18a,b) does not produce the target reading. The formal alternatives to (18a), for

[23] This replicates what Crnič et al. (2015) found for sentences of the form $\forall x(Px \vee Qx)$.

[24] See Crnič et al. (2015) for a proposal that derives the parallel inferences $\exists x(Px)$ and $\exists x(Qx)$ from $\forall x(Px \vee Qx)$ from global+embedded exhaustification.

example, which we may abbreviate as $\exists(\Diamond(P \vee Q))$, are of the form $\exists x(\Diamond Px)$, $\exists x(\Diamond Qx)$, $\exists(\Diamond(P \wedge Q))$. Of these, only the conjunctive alternative is excludable, and although the remaining alternatives $\exists x(\Diamond Px)$, $\exists x(\Diamond Qx)$ are includable, their inclusion produces the inference that some girl(s) is/(are) allowed to eat ice cream, and that some girl(s) is/(are) allowed to eat cake. This does not imply that there are girls who have free choice between the two.[25] The same point can be made about (17).

From this, I conclude that Bar-Lev and Fox's account of embedded FC from global exhaustification is incomplete. In the case of FC under *want*, *believe*, etc., the account relies on the availability of potentially problematic formal alternatives; in the case of FC under possibility expressions and existential quantifiers, the account falls short of generating the target readings.

7 Conclusions and remaining issues

My claims in this paper were based on what appear to be embedded occurrences of FC inferences. I discussed the challenge that the data present to certain views of exhaustification, specifically Del Pinal et al. (2022) and Bar-Lev and Fox (2020). On the former view, the empirical challenges become instances of familiar problems of presupposition projection, and while the account seems to offer some steps towards capturing the data correctly, it relies on assumptions that remain to be confirmed concerning presupposition projection, and in some cases (e.g., the scope of *need*) it generates unattested inferences. On the latter account, the target inferences require global exhaustification with inclusion (alongside exclusion), and generating the inferences requires auxiliary assumptions about formal alternatives that (I argued) are unmotivated.[26]

Where does this leave us? As I said earlier, I do not intend this paper to provide an argument in favor of any particular view of exhaustification. The presentation is instead aimed at identifying some of the challenges that come up if embedded FC (and FC generally) is to be treated as a scalar implicature. I note also that the findings are relevant to Goldstein's (2019) account of FC, which (like the Pex-based account) generates FC from the disjunctive form $\Diamond(p \vee q)$ along with homogeneity $(\Diamond p \leftrightarrow \Diamond q)$, and assigns homogeneity the status of a presupposition. Goldstein sees the homogeneity of FC disjunction to be similar to the homogeneity of conditionals (von Fintel 1997) and in some respects like definite plurals (Križ 2015). These do not

[25]We may add to our existential alternatives the results of replacing *some/at least one* with the universal quantifier. This gives us $\forall x(\Diamond Px)$, $\forall x(\Diamond Qx)$, $\forall(\Diamond(P \wedge Q))$. These alternatives are excludable, but their exclusion does not lead to the target existential FC inference.

[26]Focusing on the comparison between these two views, it is worth remarking that, on the Pex-based account of FC, it is not strictly necessary to use the notion of inclusion. The reader may verify this by observing that for any propositional argument ϕ and set of alternatives C, $\mathrm{Pex}_C^{\mathrm{E+I}}(\phi)$ is equivalent to recursive application of the exclusion-only $\mathrm{Pex}_C^{\mathrm{E}}$ to ϕ with intervening accommodation. That is, $\mathrm{Pex}_C^{\mathrm{E+I}}(\phi)$ is equivalent to $\mathrm{Pex}_B^{\mathrm{E}}(\mathrm{Acc}(\mathrm{Pex}_C^{\mathrm{E}}(\phi))$—here B is the set of alternatives to $\mathrm{Acc}(\mathrm{Pex}_C^{\mathrm{E}}(\phi))$, generated in the same way as the alternatives to the higher occurrence of $\mathrm{Exh}^{\mathrm{E}}$ on Fox's account of FC. To the extent that such a theoretical move is possible, the empirical motivation for adding inclusion to exhaustification is weakened. (I thank Guillermo del Pinal for discussion of this point.)

project like other presupposition triggers, so a thorough evaluation of the proposal (given our data) would have to involve looking at how the homogeneity of (e.g.) conditionals projects through the intensional contexts that we looked at, and how that compares to the homogeneity of FC disjunctions. I leave this comparison to a future occasion.

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