Remarks on disjunctive focus-associates[1]

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Abstract. This paper concerns similarities and differences between the implicatures of disjunction — modalized disjunction specifically — and the implicatures of other scalar terms like *some*. The two inference types are compared with special attention to their behavior under association with *only*, where they are shown to come apart. I argue that disjunct alternatives (p, q) to a disjunctive associate of *only* ([p or q]) do not feed the semantics of the particle. The significance of the claim is discussed in connection to (a) semantic and implicature-based accounts of free choice inferences, and (b) the phenomenon of presupposed implicatures.

Keywords: Free choice permission, disjunction, *only*, exhaustification, innocent exclusion, innocent inclusion, focus-marking, focus-alternatives.

The proposal that free choice inferences of disjunction are implicatures (Kratzer and Shimoyama 2002; Alonso-Ovalle 2005; Fox 2007, a.o.) is by no means problem-free. For one thing, familiar implicatures are cancellable with *in fact* sequences, but free choice (FC) is not (Barker 2010):

(1) a. M ate some of the cookies. In fact, s/he ate them all.
   b. M is allowed to eat cake or ice cream. #In fact, s/he is allowed to eat just cake.

From an implicature-based perspective of FC, (1) is indeed surprising. However, it isn't obvious how much of the contrast has to do with whether FC is an implicature, and how much has to do with the properties of *in fact* locutions. In the absence of a theory of the latter, there is no reason to think that (1) constitutes an insurmountable challenge.[2]

The trouble does not stop there, however. In the antecedent of a conditional the FC inference is more easily detectable than the strengthened reading of *some*. Consider first (2):

(2) If M tries some of grandma's food, grandma will be disappointed.

If it were possible to exhaustify *some* inside the *if*-clause in (2), the sentence would allow the reading that grandma will be disappointed if M tries some but not all of the food. The sentence should then be true when it is said about grandmothers (like mine) who enjoy making food, and enjoy seeing all of it be eaten. But without any marked prosody (2) cannot be understood this

[1]Many — but not all — of the points discussed in this paper were published in Alxatib 2020, a paper that was still under review at the time of submission to *Sinn und Bedeutung* 25. To minimize duplication, I turned my attention here to a number of issues that I did not address in Alxatib 2020. For comments and discussion, I thank the anonymous reviewers for *Sinn und Bedeutung 25* and *Natural Language Semantics*, Luka Crnič, Natasha Ivlieva, Paloma Jeretič, Filipe Hisao Kobayashi, Angelika Kratzer, Paul Marty, Andreea Nicolae, Jacopo Romoli, Philippe Schlenker, Yael Sharvit, Yasutada Sudo, audiences at Philippe Schlenker's Formal Pragmatics seminar, the 94th LSA meeting, and *Sinn und Bedeutung 25*.

[2]One possibility is that *in fact* targets only scalar alternatives rather than subdomain alternatives (see e.g. Chierchia 2013 on this distinction).

 In: P. G. Grosz, L. Martí, H. Pearson, Y. Sudo, and S. Zobel (eds.) *Proceedings of Sinn und Bedeutung 25*, pp. 37–54. University College London and Queen Mary University of London.

way; it suggests that grandma will be disappointed if M tries *any* of her food. Therefore *some*, without accenting, cannot be strengthened inside an antecedent of a conditional.

By contrast, FC readings are available inside the antecedent of a conditional, as Kamp (1978) has pointed out: (3) can mean that M will be pleased because s/he is free to choose between cake and ice cream.

(3) If M is allowed to eat cake or ice cream, she will be pleased.

And the following interactions suggest the same conclusion:

(4) A: If you are allowed to either try the cake or the ice cream, write your name down here.
B: I am allowed to try the cake. Should I write my name down?[3]

(5) A: If you tried some of the desserts, write your name down here.
B: ??I tried all of the desserts. Should I write my name down?

B's question in (4) is understandable, since there is a way to take A's directive to be about people who, unlike B, are free to choose between cake and ice cream. But in (5), B's question seems tedious without additional context or special accenting on *some* in A's utterance. Once again, FC appears to behave differently from *some*'s scalar implicature.

Here is a third difference: when a scalar item like *some* is focused under *only*, the 'not all' inference becomes part of *only*'s assertion (6); but when a disjunctive phrase is focused under *only*, as in (7), the FC inference is presupposed, not asserted (Alxatib 2014).

(6) a. M only ate some$_F$ of grandma's food.
Presupposes M ate some of grandma's food;
Asserts that M did not eat more than some (not a lot; not all; etc).
b. Did M only eat some$_F$ of grandma's food?
Presupposes M ate some of grandma's food;
Inquires whether M ate more than some (not a lot; not all; etc).

(7) a. M is only allowed to eat [cake or ice cream]$_F$.
Presupposes FC;
Asserts that M is not allowed to eat other desserts.
b. Is M only allowed to eat [cake or ice cream]$_F$?
Presupposes FC;
Inquires whether M is allowed to eat other desserts.

Why is FC presupposed in (7)?

A proponent of the semantic view of FC disjunctions may say that they have the answer.[4] FC is not an implicature; it is the product of composing the meanings of disjunction with the

[3]Following Larson 1985, I use dislocated *either . . . or* in (4) to try and ensure that disjunction take scope below the modal.

[4]No one has made this argument on the basis of (7) to my knowledge, but on first sight it is inviting given the contrast with (6). Examples of semantic accounts of FC include Simons 2005, Aloni 2007, and Goldstein 2019.

meanings of (at least some) modal expressions. And because *only* generally presupposes its prejacent (Horn 1969 and many since), it follows that FC will be presupposed in (7).

This response explains the other findings too: because FC is part of the semantics, it should be detectable inside *if*-clauses, and it should resist cancellation by *in fact*.

Alternatively we may follow Bar-Lev and Fox (2020) and use the distinction that they propose between excludable formal alternatives and includable formal alternatives. In the case of *only* and its FC presupposition, we can say (as Bar-Lev and Fox propose) that *only* presupposes not just its prejacent but its prejacent and its includable alternatives. These, in the case of (7), are the alternatives in which the focused disjunction [*cake or ice cream*] is replaced with its disjuncts *cake* and *ice cream* — I will explain this later. If the semantics of *only* were such that it presupposed these alternatives, then in (7) the particle will presuppose that M is allowed to eat cake — from the includable alternative [*M is allowed to eat cake*] — and that she is allowed to eat ice cream — from the includable alternative [*M is allowed to eat ice cream*].

I will argue that accepting either of these possibilities on the basis of (7) is hasty. My argument will be built on comparing (7) to cases like it but where a necessity modal takes the place of the possibility modal, like (8):

(8) a. M is only required to eat [cake or ice cream]$_F$.
Presupposes M is not required to eat cake, and not required to eat ice cream;
Asserts that M is not required to eat other desserts.

b. Is M only required to eat [cake or ice cream]$_F$?
Presupposes M is not required to eat cake, and not required to eat ice cream;
Inquires whether M is required to eat other desserts.

Only's focus associate in (8) is disjunctive, so we expect the prejacent to have — as it does in (7) — alternatives where the disjuncts replace the disjunction. But in (8), unlike the case of (7), these alternatives are <u>ex</u>cludable given *only*'s prejacent: their negations are jointly consistent with it. We therefore expect *only* to <u>assert</u> their falsity, not to presuppose it. Yet the judgement shows clearly that the inference is presupposed. Why is that?

Let us remind ourselves that without *only*, a necessity modal that outscopes disjunction robustly licenses the "distribution" inference that (8) presupposes. (9) shows this.

(9) M is required to eat cake or ice cream.
$\rightsquigarrow$ M is not required to eat cake, and not required to eat ice cream

So now we have a similar challenge to the one from (7). If the inference from $\Box(p \vee q)$ to $\neg\Box p, \neg\Box q$ is an implicature, then why is the inference not asserted upon association with *only* in (8)? Can it be that the inference is not an implicature in this case either? It may be tempting to say yes, but this time I think there is a reason not to. Look at (10):

(10) A: If you are required to either try the cake or the ice cream, write your name down here.

B: #I am required to try the cake. Should I write my name down?

B's response in (10) is strange, but it shouldn't be if A's instruction were literally about those who must try one of the two desserts but neither one specifically. On the other hand the judgement makes sense if the "distribution" inference of $\Box(p \vee q)$ is an implicature, since it would then match the effect we found with *some* in (5).

(11) and (12) also show the difference, more clearly I think: (11) can describe a situation where three people have free choice and a fourth person can only eat cake; but (12) cannot describe a situation where three people (each) have the more relaxed requirement of eating one or the other dessert (but neither specifically), while a fourth person has the stricter requirement to eat cake.

(11) Not everyone is allowed to eat cake or ice cream.

(12) Not everyone is required to eat cake or ice cream.

Once again we see evidence that the inference from $\Box(p \vee q)$ to $\neg\Box p, \neg\Box q$ is not part of the literal semantics of $\Box(p \vee q)$, and from this point forward I will assume that the inference is indeed an implicature. What remains is (i) to explain why the inference is presupposed, rather than asserted, upon association with *only*, and (ii) to think about how this helps us understand the parallel case of the possibility modal — (7).

Pursuing (i), as I will do in the next few pages, requires that we look at the semantics of *only*, and in particular the way that it interacts with disjunctive focus associates. There is an important and repeatedly noted guiding concern in studying this interaction: any two propositions p, q are (each) logically stronger than their disjunction $p \vee q$. So if a disjunctive focus associate like the one in (13) has its disjuncts as formal alternatives, then we do not want to write a semantics of *only* that negates all of the associate's stronger (or non-weaker) alternatives; negating the (stronger/non-weaker) disjuncts would contradict the disjunction.

(13) M only ate [salad or soup]$_{\mathrm{F}}$

Here is an entry that avoids the contradiction, based on van Rooij and Schulz 2007. Suppose C is a set of propositions. Then, let us write $C|_w$ to mean the subset of C that consists of true propositions in w. That is:

(14) $C|_w = \{\psi : \psi \in C \;\&\; \psi(w) = 1\}$

With this, we can say that $[\![only]\!](C)(\phi)(w)$ presupposes that ϕ is an element of $C|_w$ — this is *only*'s prejacent presupposition — and asserts that it is not possible for $C|_w$ to be any smaller, that is, that there are no possible worlds v where $C|_v$ also contains ϕ but where $C|_v$ is a proper subset of $C|_w$:

(15) <u>***Only* (adapted from van Rooij and Schulz)**</u>:
$[\![only]\!](C)(\phi) = [\lambda w : \phi \in C|_w \,.\, \neg\exists v(\phi \in C|_v \;\&\; C|_v \subset C|_w)]$

When $\phi = (p \vee q)$ and $C = \{p \vee q, p, q, p \wedge q\} / C = \{p \vee q, p, q, r\}$, $[\![only]\!](C)(\phi)$ will presuppose $(p \vee q)$ and assert that no other alternative is true aside from p or q. This is because the smallest subsets of C whose elements are all true and which include the disjunction $(p \vee q)$ must contain either p or q; the disjunction cannot be true otherwise. But any world that verifies more than the elements of $\{p \vee q, p\}$ or $\{p \vee q, q\}$ from C will fail the condition, because it is logically possible for these propositions to be true without the remaining members of C.[5]

Now, what about the case of $\phi = \Box(p \vee q)$, $C = \{\Box(p \vee q), \Box p, \Box q, \Box(p \wedge q)\}$? Here, logical consistency allows for ϕ to be true alone among its set-mates. Therefore we make the same incorrect prediction observed earlier: constructions of the form $only_C(\Box(p \vee q)_\mathrm{F})$ should presuppose $\Box(p \vee q)$ and <u>assert</u> $\neg\Box p, \neg\Box q, \neg\Box(p \wedge q)$, etc. This is incorrect, recall, because intuitively these inferences are presupposed:

(8) a. M is only required to eat [cake or ice cream]$_\mathrm{F}$.
Presupposes M is not required to eat cake, and not required to eat ice cream;
Asserts that M is not required to eat other desserts.

b. Is M only required to eat [cake or ice cream]$_\mathrm{F}$?
Presupposes M is not required to eat cake, and not required to eat ice cream;
Inquires whether M is required to eat other desserts.

This prediction is inherited by every account (that I know of) that solves the problem of disjunction.[6] And the reason is more or less the same across the board: Each account permits a presupposition-assertion combination as long as it isn't contradictory, and in cases like (8) the falsity of the disjunct alternatives is consistent with the disjunctive prejacent.[7]

Now is a good time to turn to Bar-Lev and Fox's (2020) proposal. As I said above, the account incorporates Fox's (2007) notion of innocent excludability as well as the novel notion of innocent includability into the meaning of *only* (see also Bar-Lev 2018). The proposal has it that *only* presupposes its prejacent's includable alternatives from C, and asserts the negations of its excludable ones. The notions will be important throughout the paper, so I will explain them in the next few paragraphs. Let me note once again, however, that as far as (8) goes

[5]Making use of logical possibility is the reason behind quantifying over possible worlds in (15). Notice that the quantification makes no reference to an accessibility relation from w, so the entry is not a commitment to a modal semantics of *only*. Another note about the technical detail: I am assuming that any expression is a formal alternative of itself; things can be recast without this assumption, however.

[6]Fox (2003) uses Kratzer's (1989) notion of lumping in his account of *only* to solve the issue with disjunction (see also Heim 1990). However, the mere fact that $\Box(p \vee q)$ can be true without the truth of either $\Box p$ or $\Box q$ tells us that $\Box(p \vee q)$ does not lump $\Box p, \Box q$. Therefore limiting *only*'s exclusions to propositions not lumped by the prejacent will have the same unwanted effect: it will make *only* assert $\neg\Box p, \neg\Box q$ when it takes $\Box(p \vee q)_\mathrm{F}$ as prejacent. And Bonomi and Casalegno (1993), who do not talk about disjunction directly and do not talk about modals at all, use event(tuality) parthood in their semantics of *only* (Heim (1990) discusses this possibility too, though briefly). To Bonomi and Casalegno, *only*'s assertion says that all eventualities that exemplify an alternative to the prejacent are parts of the (relevant) eventuality that exemplifies the prejacent. I do not know confidently what it takes for an eventuality of obligation/permission to part of another, but again, the intuition that $\Box(p \vee q)$ can hold without $\Box p, \Box q$ is enough to tell us that e can make $\Box(p \vee q)$ true without having any "parts" in which $\Box p, \Box q$ are true. The account is therefore equally susceptible to the problem in (8).

[7]I do not know yet if this is true of movement-based views of focus association (e.g. Chomsky 1976; Drubig 1994; Wagner 2006). We can imagine that on theories of this kind, the disjunctive associate in cases like (8) takes scope above the necessity modal, in an LF that looks like this: [[*only* [p or q]$_\mathrm{F}$] [λ_1 ... $\Box$ ... t_1]]. Assuming such an LF, the disjunction will interact with *only* in the same way that it does when unmodalized, so we correctly keep the disjunct alternatives away from *only*'s reach. However, we also produce the incorrect presupposition that one of the two disjuncts is required. This is not the target meaning of (8). I leave investigation of this, and related possibilities, to a future occasion.

the unwanted prediction will be the same: the inferences that are intuitively presupposed are incorrectly generated in *only*'s assertion.

Innocent exclusion. An alternative ψ from a set C is innocently excludable (IE) given ϕ iff every selection of exclusions from C that is inconsistent with ϕ remains inconsistent with it without $\neg\psi$. A "selection of exclusions from C" is a conjunction of the negations of some set of elements of C; that is, a proposition of the form $\neg\psi_1$ & $\neg\psi_2$ & $\cdots$ & $\neg\psi_n$ for some $\psi_1, \psi_2, \cdots, \psi_n$ that appear in C. Sometimes such a selection contradicts ϕ, the proposition whose formal alternatives populate C. But for any such selection of exclusions we can identify the culprits, i.e. the alternatives whose negations are responsible for the contradiction. If removing an exclusion $\neg\psi_i$ prevents the contradiction, then ψ_i is complicit in it; if not, it is not. An innocently excludable alternative is an alternative that is never complicit in this respect: any selection of exclusions that includes its negation, and that contradicts ϕ, is a selection that contradicts ϕ anyway, without the given alternative's negation.

Here is the definition in more precise terms:

(16) **Innocent Excludability**:
Given a proposition ϕ and set of propositions C, ψ is innocently excludable (IE) given C and ϕ iff $\psi \in C$ and $\forall B(B \subseteq C \ \& \ \phi \wedge \bigwedge B^{\neg} \models \bot \rightarrow \phi \wedge \bigwedge(B - \{\psi\})^{\neg} \models \bot)$
Where for any set of propositions B, $B^{\neg} = \{\chi : \exists\psi(\psi \in B \ \& \ \chi = \neg\psi)\}$

By this definition, the disjuncts p, q of a disjunction $\phi = (p \vee q)$ are not IE with respect to ϕ (assuming $C = \{p \vee q, p, q, p \wedge q\}$). This because the negations of elements of $B = \{p, q\}$ — a subset of C — jointly contradict the disjunction, but if either element is removed from B, the negations of what remains are consistent with the disjunction. Therefore neither p nor q is innocently excludable. On the other hand the conjunctive alternative $(p \wedge q)$ is innocently excludable, because it never affects the (in)consistency of any selection of exclusions with ϕ. Alternatives that are independent of p and q are also predicted to be innocently excludable. So:

(17) Given $\phi = (p \vee q), C = \{p \vee q, p, q, r, p \wedge q\}$,
$\mathrm{IE}(C)(\phi) = \{r, p \wedge q\}$

In the case of $\phi = \Box(p \vee q)$ and $C = \{\Box p, \Box q, \Box(p \wedge q)\}$, all the elements of C are predicted to be IE, because they can all be negated consistently with ϕ. In terms more closely connected to the definition above: there are no selections of exclusions that contradict ϕ, so every element of C satisfies the condition trivially:

(18) Given $\phi = \Box(p \vee q), C = \{\Box(p \vee q), \Box p, \Box q, \Box r, \Box(p \wedge q)\}$,
$\mathrm{IE}(C)(\phi) = C = \{\Box(p \vee q), \Box p, \Box q, \Box r, \Box(p \wedge q)\}$

Let us abstract away from these technical details and define a proposition $\textit{\textbf{exclusions}}_C(\phi)$. This is simply the conjunction of the exclusions (negations) of whatever can be excluded from ϕ's alternatives in C. To Bar-Lev and Fox, these are the IE alternatives:

(19) Given a proposition ϕ and set of propositions C,
$\textbf{\textit{exclusions}}_C(\phi) = [\lambda w\,.\,\forall\psi(\psi\in\mathrm{IE}(C)(\phi) \rightarrow \psi(w)=0)]$

Now we can write the following (preliminary) semantics for *only*: the particle presupposes its prejacent (to be reconsidered) and asserts the prejacent's exclusions:

(20) $[\![only]\!](C)(\phi) = [\lambda w : \phi(w)=1\,.\ \textbf{\textit{exclusions}}_C(\phi)(w)]$

For an unmodalized disjunctive associate, (20) produces the exclusive inference and the negation of other independent alternatives. For a disjunctive associate across a necessity modal, we get the noted incorrect assertion that the disjunct inferences are false:

(21) If $\phi=(p\vee q)$ and $C=\{p\vee q, p, q, r, p\wedge q\}$, then
$[\![only]\!](C)(\phi) = [\lambda w : [p\vee q](w)=1\ .\ r(w)=0\ \&\ [p\wedge q](w)=0]$

(22) If $\phi=\Box(p\vee q)$ and $C=\{\Box(p\vee q), \Box p, \Box q, \Box r, \Box(p\wedge q)\}$, then
$[\![only]\!](C)(\phi) = [\lambda w : [\Box(p\vee q)](w)=1$
$.\ [\Box p](w)=0\ \&\ [\Box q](w)=0\ \&\ [\Box r](w)=0\ \&\ [\Box(p\wedge q)](w)=0]$

(When I return to the unwanted assertion of $\neg\Box p, \neg\Box q$, I will claim that *only* does not see disjunct alternatives at all. If this is correct, two things follow: (i) Bar-Lev and Fox's account of *only*, and specifically of its FC presuppositions, is incorrect; (ii) if $\neg\Box p, \neg\Box q$ are indeed implicatures of $\Box(p\vee q)$ — see justification above — and if they are presupposed in cases like (8/22), then we have independent evidence that disjunctive associates of *only* can license *presupposed implicatures*. This brings the FC presupposition under *only*, in e.g. (7), within reach again for implicature views of FC.)

Let us turn now to Bar-Lev and Fox's account of *only*'s FC presupposition. Here, again, is the example:

(7) a. M is only allowed to eat [cake or ice cream]$_\mathrm{F}$.
Presupposes FC;
Asserts that M is not allowed to eat other desserts.

b. Is M only allowed to eat [cake or ice cream]$_\mathrm{F}$?
Presupposes FC;
Inquires whether M is allowed to eat other desserts.

Innocent inclusion. Let me begin by introducing the unpronounced operator Exh, which I will assume generates implicatures. For now we say that Exh asserts its prejacent and its exclusions:

(23) $[\![\mathrm{Exh}]\!](C)(\phi) = [\lambda w : \phi(w)=1\ \&\ \textbf{\textit{exclusions}}_C(\phi)(w)=1]$

We determined earlier that p, q are not IE given $\phi=(p\vee q)$ and $C=\{p\vee q, p, q, r, p\wedge q\}$, and that $\Box p, \Box q$ <u>are</u> IE (as is every other member of C) given $\phi=\Box(p\vee q)$ and $C=\{\Box(p\vee q), \Box p, \Box q, \Box r, \Box(p\wedge q)\}$. Exhaustification in these two cases therefore gives us the following results (paralleling the exclusions of *only*):

(24) When $\phi = (p \vee q)$, $C = \{p \vee q, p, q, r, p \wedge q\}$,
$[\![\text{Exh}]\!](C)(\phi) = [\lambda w : [p \vee q](w) = 1 \ \&\ r(w) = 0 \ \&\ [p \wedge q](w) = 0]$

(25) When $\phi = \Box(p \vee q)$, $C = \{\Box(p \vee q), \Box p, \Box q, \Box r, \Box(p \wedge q)\}$,
$[\![\text{Exh}]\!](C)(\phi) = [\lambda w : [\Box(p \vee q)](w) = 1 \ \&\ [\Box p](w) = [\Box q](w) = [\Box r](w) = 0]$

With possibility modals the result is like that of unmodalized disjunction: given $\phi = \Diamond(p \vee q)$ and $C = \{\Diamond(p \vee q), \Diamond p, \Diamond q, \Diamond r, \Diamond(p \wedge q)\}$, the disjunct alternatives $\Diamond p, \Diamond q$ are not innocently excludable. In this case exhaustification gives us the following incomplete result — 'incomplete' because we have yet to derive FC as an implicature:

(26) When $\phi = \Diamond(p \vee q)$, $C = \{\Diamond(p \vee q), \Diamond p, \Diamond q, \Diamond r, \Diamond(p \wedge q)\}$,
$[\![\text{Exh}]\!](C)(\phi) = [\lambda w : [\Diamond(p \vee q)](w) = 1 \ \&\ [\Diamond r](w) = 0 \ \&\ [\Diamond(p \wedge q)](w) = 0]$

Inclusions are positive counterparts to exclusions: they are inferences of the form 'ψ is true' rather than 'ψ is false', ψ being a suitable formal alternative. An alternative ψ is innocently includable given ϕ and set of alternatives C iff every selection of *in*clusions from C that contradicts ϕ & $\textit{\textbf{exclusions}}_C(\phi)$ remains inconsistent with it without ψ. By a "selection of inclusions from C" I mean a conjunction of some set of elements of C, that is, a proposition of the form $\psi_1 \ \&\ \psi_2 \ \&\ \cdots \ \&\ \psi_n$ for some $\psi_1, \psi_2, \cdots, \psi_n \in C$. An innocently includable alternative ψ is one that is never complicit in making such a selection contradict ϕ & $\textit{\textbf{exclusions}}_C(\phi)$: any selection of inclusions that features ψ, and that contradicts ϕ & $\textit{\textbf{exclusions}}_C(\phi)$, is a selection that contradicts ϕ anyway, that is, without ψ.

We are about to see that inclusions add the FC inference $\Diamond p, \Diamond q$ to (26), as desired, and that they add no unexpected implicatures to (24,25), also as desired. The definition of innocent includability is stated in (27):

(27) **<u>Innocent Includability</u>**:
Given a proposition ϕ and set of propositions C, ψ is innocently includable (II) given C and ϕ iff $\psi \in C$ and $\forall B(B \subseteq C \ \&\ \phi \ \&\ \textit{\textbf{exclusions}}_C(\phi) \wedge \bigwedge B \vDash \bot$
$\rightarrow (\phi \ \&\ \textit{\textbf{exclusions}}_C(\phi)) \wedge \bigwedge (B - \{\psi\}) \vDash \bot)$

Let us determine first the II alternatives from C in (25). In this case, every element of C aside from $\phi = \Box(p \vee q)$ is innocently excludable, and therefore participates in $\textit{\textbf{exclusions}}_C(\phi)$. That leaves ϕ as the only alternative that can be asserted consistently with ϕ & $\textit{\textbf{exclusions}}_C(\phi)$; every other element of C is negated in $\textit{\textbf{exclusions}}_C(\phi)$. The set of II alternatives in this case is therefore the singleton $\{\Box(p \vee q)\}$.[8]

With plain disjunction ($\phi = (p \vee q)$) the set of II alternatives also contains nothing but ϕ. The reason here, however, isn't as trivial as in case of $\Box(p \vee q)$. Consider again the set $B = \{p, q\}$, which is a subset of $C = \{p \vee q, p, q, p \wedge q\}$. Asserting the elements of B alongside $(p \vee q)$ <u>and its exclusions</u> is contradictory, because the exclusions negate the conjunctive alternative $(p \wedge q)$. But if we remove q from B, this changes, as it does if we remove p instead. Therefore each of p, q is complicit in making B contradict ϕ & $\textit{\textbf{exclusions}}_C(\phi)$ in this case.

[8]On the assumption that an expression is always an alternative of itself, it follows by definition that it will be innocently includable.

With possibility modals, however, the disjunct alternatives $\Diamond p, \Diamond q$ are predicted to be includable, because their assertions (unlike those of the previous two cases) are jointly consistent with $\phi = \Diamond(p \vee q)$ and its exclusions: $\Diamond p, \Diamond q$ are simultaneously consistent with $\Diamond(p \vee q)$ & $\neg\Diamond(p \wedge q)$.

These then are the results:

(28) When $\phi = (p \vee q), C = \{p \vee q, p, q, r, p \wedge q\}$,
$\text{II}(C)(\phi) = \{p \vee q\}$

(29) When $\phi = \Box(p \vee q), C = \{\Box(p \vee q), \Box p, \Box q, \Box r, \Box(p \wedge q)\}$,
$\text{II}(C)(\phi) = \{\Box(p \vee q)\}$

(30) When $\phi = \Diamond(p \vee q), C = \{\Diamond(p \vee q), \Diamond p, \Diamond q, \Diamond r, \Diamond(p \wedge q)\}$,
$\text{II}(C)(\phi) = \{\Diamond(p \vee q), \Diamond p, \Diamond q\}$

We are now ready to put together Bar-Lev and Fox's definitions of Exh and *only*. Let us write $\textbf{\textit{inclusions}}_C(\phi)$ as shorthand for the conjunction of ϕ's includable alternatives from C:

(31) $\textbf{\textit{inclusions}}_C(\phi) = [\lambda w \,.\, \forall \psi(\psi \in \text{II}(C)(\phi) \rightarrow \psi(w) = 1)]$

Then (going by Bar-Lev and Fox's definitions) $[\![\text{Exh}]\!](C)(\phi)$ asserts ϕ's inclusions and exclusions, (32), while $[\![\textit{only}]\!](C)(\phi)$ presupposes ϕ's inclusions and asserts its exclusions (33):

(32) **Bar-Lev and Fox's Exh**:
$[\![\text{Exh}]\!](C)(\phi) = [\lambda w \,.\, \textbf{\textit{inclusions}}_C(\phi)(w) = 1 \;\&\; \textbf{\textit{exclusions}}_C(\phi)(w) = 1]$

(33) **Bar-Lev and Fox's *only***:
$[\![\textit{only}]\!](C)(\phi) = [\lambda w : \textbf{\textit{inclusions}}_C(\phi)(w) = 1 \,.\, \textbf{\textit{exclusions}}_C(\phi)(w)]$

With (32) and (33), we are in a good place to retrace our steps and see what's coming. (32) is a view of exhaustification — scalar implicature generation — that produces FC, and (33) is a lexical treatment of *only*. Both come from Bar-Lev and Fox, and as is clear from the entries, both license the same inferences. The discussion that led us here began with a problem: how to explain the fact that FC is presupposed in (7), when in other cases *only* folds what would otherwise be implicatures into its assertion.

(7) a. M is only allowed to eat [cake or ice cream]$_F$.
Presupposes FC;
Asserts that M is not allowed to eat other desserts.

b. Is M only allowed to eat [cake or ice cream]$_F$?
Presupposes FC;
Inquires whether M is allowed to eat other desserts.

Bar-Lev and Fox's answer is that FC comes not from exclusion but from inclusion, and inclusions are presupposed by *only*, not asserted. The flipside is: all implicatures that come by way of exclusion should be asserted by *only*. But as we saw in (8), *only* intuitively presupposes what ought to be exclusions:

(8) a. M is only required to eat [cake or ice cream]$_F$.
Presupposes M is not required to eat cake, and not required to eat ice cream;
Asserts that M is not required to eat other desserts.

b. Is M only required to eat [cake or ice cream]$_F$?
Presupposes M is not required to eat cake, and not required to eat ice cream;
Inquires whether M is required to eat other desserts.

My take on these findings is the following: *only* does not operate on disjunct alternatives to a disjunctive focus, that is, when *only* has a prejacent of the form $[\cdots\ [p \vee q]_F \cdots]$, the set of focus alternatives C does not contain $[\cdots\ p \cdots]$ or $[\cdots\ q \cdots]$. I do not mean that disjuncts are not formal alternatives to a disjunction, nor that they do not participate in exhaustification. I simply mean that *only*'s domain is more constrained than previously thought: it is limited to those sets of alternatives whose elements bear the same relation to one another that they bear to the prejacent. The constraint is the following:

(34) **Constraint on *only*'s alternatives and prejacent**:
A proposition ϕ is in the domain of $[\![only]\!](C)$ only if:
$\exists \psi(\psi \in C\ \&\ \psi \neq \phi\ \&\ \psi \vDash \phi) \ \rightarrow\ \forall \psi \forall \chi(\psi, \chi \in C \rightarrow \psi \vDash \chi \vee \chi \vDash \psi)$

(34) says that if any alternative in C asymmetrically entails ϕ, then C must be ordered totally by entailment. This condition fails when the prejacent is disjunctive and C contains its disjunct alternatives, regardless of whether the disjunction is unembedded or embedded under a modal. Note that the definition also rules out the possibility of having both an independent alternative r to a disjunction and at the same time the conjunctive alternative in C: in this case, the conjunctive alternative asymmetrically entails the prejacent, but does not entail and is not entailed by the independent alternative r. It follows that a disjunctive associate to *only* can only have independent alternatives in C, or else have just the conjunctive alternative by itself. I think a case can be made for this result. For example in (35), without emphatic prosody on *or* in A, B1 is odd:

(35) A: Did M only eat [cake or ice cream]$_F$?
B1: ??No, s/he ate both.
B2: No, s/he (also) ate pie.

If conjunctions could, without constraint, be alternatives to disjunctive associates to *only*, then B1's response should do just as well as B2's. But there is a clear contrast between the two. On the other hand, notice that adding prosodic prominence on *or* changes the pattern:

(36) A: Did M only eat cake OR$_F$ ice cream?
B1: No, s/he ate both.
B2: ?No, s/he (also) ate pie.

The contrast between B1 and B2 may not be as strong here, but notice first that B1's answer is perfectly acceptable, unlike in (35). Notice also that in (36) even B2's answer implies that M ate both cake and ice cream, unlike B2's answer in (35). I do not have a complete account of these data, but (35) suggests fairly clearly that conjunctions are not as good formal alternatives to disjunctive associates (to *only*) as independent alternatives. This follows from (34). With focus on the connective itself, however, only the *and*-alternative remains in C, and we derive the acceptability of B1's answer in (36).[9] The acceptability of B2's "no" answer in that case remains to be explained.[10]

For our purposes the advantage of (34) is that it removes the alternatives $\Box p, \Box q$ from *only*'s reach in the case of (8). The exclusions $\neg\Box p, \neg\Box q$ are no longer predicted to be part of *only*'s assertion. This is good, but it leaves unanswered the question of where the presuppositions come from.

The parallel consequence of (34) on the case of (7) is that it makes Bar-Lev and Fox's account of *only* ineffective, for if indeed the disjunct-alternatives $\Diamond p, \Diamond q$ are blocked from membership in C, there will be no II alternatives for *only* to presuppose except the prejacent. We therefore have no account of (7)'s FC presupposition either.

It should now be clear that our target is the same for both of these cases: we want the implicatures involving the disjuncts (whether they are exclusions or inclusions) to be presupposed under *only*, despite the tendency of the particle to turn implicatures into assertions.[11] What makes *only* behave differently with disjunctive focus-associates is due to what *only* requires of its prejacent's alternatives, namely that they be ordered in a particular way. In the case of e.g. *some* and *all*, the requisite ordering relation is met, and *only* therefore negates *all* when its associate is the (weaker) *some*. Likewise in the case of a focused *or* and a conjunctive alternative. But when an entire disjunctive phrase is focused, C must consist of nothing but independent alternatives.

How then do we derive implicatures as presuppositions? This question has been brought up before, notably by Gajewski and Sharvit (2012) and since then by Marty and Romoli (2020).[12] As a first example, take (37).

(37) J is unaware that some of the dessert was eaten.

The verb *unaware* has both a presupposition and an assertion. Its presupposition is factive, and its assertion is about its subject's belief state. In (37) the verb embeds a clause that contains

[9]Notice that, in principle, C could also have just the conjunctive alternative even without prosody on *or*. So then why is B1's response strange in (35)? Perhaps in such cases, a principle like Schwarzschild's (1999) AvoidF requires that focus be as broad as necessary and no broader. If the only alternative in C varies by just the connective, then focus should be placed on just the connective.

[10]Paloma Jeretič (p.c.) suggested to me that (36B2) may be acceptable because it answers the question and supplements it with additional information: 'no [*i.e. M ate cake* <u>*and*</u> *ice cream*], [*and moreover*] s/he also ate pie'. If this is right, then we expect the answer to become worse — or at least sound like a rejection of A's prejacent presupposition — without *also*, since that would make B2's continuation feel more like an explanation of their "no" answer than an addition to it. I think the judgement below confirms this, but I leave the matter to future study.

(i) A: Did M only eat cake OR_{F} ice cream?

B: [??]No, she ate pie.

[11]See e.g. Fox's (2003) and Fox and Hackl's (2006) *only*-implicature generalization.

[12]See also Simons 2006 and Russell 2006 for earlier discussions of related findings, and see Spector and Sudo 2017 for a critique of Gajewski and Sharvit.

the scalar term *some*, but interestingly, the contribution of *some* is enriched in the verb's factive presupposition alone, not in the assertion. The sentence, in other words, is appropriate only in contexts where some but not all of the dessert was eaten — enriched presupposition — and is true just in case J considers it possible that the dessert remains uneaten, that is, that the plain existential meaning of *some* is false.

We reach a similar point from (38):

(38) Only $\mathrm{J_F}$ did some of the homework.
Presupposes: J did some but not all of the homework
Asserts: No one other than J did any homework

Here, again, *some* is enriched in the (positive) presupposition alone — not in the (negative) assertion — but this time we see the effect in the presence of *only* and, crucially, with *some* unfocused. Putting *some* in focus, as we have seen, feeds *only*'s assertion:

(39) J only did $\mathrm{some_F}$ of the homework.
Presupposes: J did some of the homework
Asserts: J did not do more than some of the homework

The difference between (38) and (39) — with respect to whether *some*'s enrichment is asserted or presupposed — may appear to be determined by focus-marking. But we can characterize the difference in another way: the enrichment of a scalar term in *only*'s prejacent feeds the presupposition, rather than the assertion, when the enrichment is based on alternatives that do not appear in *only*'s C set. In (38), C does not include the *all*-alternative to the prejacent, because *some* is not focused. But being unfocused is not the only way for a formal alternative to be missing from C: by the constraint in (34), the disjunct-alternatives to a focused disjunction will also be missing from *only*'s C set, despite the fact that the disjunction carries focus. Therefore the enrichment that uses the disjuncts in such cases is added to the prejacent presupposition, not to *only*'s assertion. It is this, I propose, that generates the FC presupposition in (7) and the distributive inference in (8), repeated below:

(7) M is only allowed to eat [cake or ice cream]$_\mathrm{F}$.

(8) M is only required to eat [cake or ice cream]$_\mathrm{F}$.

Before I turn to the technical details, I will (again) risk laboring the point and remind the reader of the significance of (8). The distribution inference $\neg\Box p, \neg\Box q$, recall, is not likely the result of a *semantic* interaction between necessity modals and disjunction (see (10) above). And we have seen that the presupposition in (8) is not derivable on Bar-Lev and Fox's account of *only*. What we learn from the example, I think, is that disjunctive foci interact in a restricted way when under association with *only*. This, in turn, makes an implicature-based account of the FC presupposition in (7) more plausible, since we now *expect* the FC implicature to be presupposed, not asserted.

Now the details. To Gajewski and Sharvit, the presuppositional enrichment in cases like (37) shows that implicatures can be calculated in the presuppositional tier independently of assertions. And although the picture turns out to be more complicated — as Spector and Sudo and

Marty and Romoli have argued — for our purposes we can skirt the complications and follow Gajewski and Sharvit's basic idea.[13] Here is a first formulation: exhaustifying a proposition ϕ enriches both its domain and its output. The enrichment of ϕ's domain (the presupposition) comes from exhaustifying that domain against the domains of ϕ's alternatives. In other words: assuming (i) that ϕ presupposes ψ, (ii) that C consists of ϕ's alternatives, (iii) that C^{Dom} is the set of propositions presupposed by the elements of C (the domains of C's elements), then the domain of ϕ is enriched by applying $[\![\text{Exh}]\!]$ to ψ, given C^{Dom} as a set of alternative propositions:

(40) <u>**Definedness condition on Exh (Take 1)**</u>:
$Dom([\![\text{Exh}]\!](C)(\phi)) \subseteq [\![\text{Exh}]\!](C^{Dom})(Dom(\phi))$

Or alternatively we can write:

(41) $[\![\text{Exh}]\!](C)(\phi) = [\lambda w : [\![\text{Exh}]\!](C^{Dom})(Dom(\phi))(w) = 1 \,.\, \cdots]$

Let us apply (41) to (37), assuming that (37) has the formal alternative in (42), where *all* replaces *some*:

(42) J is unaware that all of the dessert was eaten.

With $\phi = unaware(\exists)$ — a schematization of (37) — and $C = \{unaware(\exists), unaware(\forall)\}$, we get $Dom(\phi) = \exists$ and $C^{Dom} = \{\exists, \forall\}$. So upon exhaustification, the domain of (37) is restricted to the worlds in which $\text{Exh}(\{\exists,\forall\})(\exists)$ holds, that is, worlds in which some but not all of the dessert was eaten.

In (38), *some* accompanies *only* but not as its focus associate. In this case I also assume that the sentence has an *all*-alternative (43), which restricts its presupposition in the same way, as summarized in (44):

(43) Only J_F did all of the homework.

(44) a. $Dom([\![\text{only } \text{J}_\text{F} \cdots \text{ some} \cdots]\!]) = \exists$
b. $Dom([\![\text{only } \text{J}_\text{F} \cdots \text{ all} \cdots]\!]) = \forall$
c. $C = \{[\![\text{only } \text{J}_\text{F} \cdots \text{ some} \cdots]\!], [\![\text{only } \text{J}_\text{F} \cdots \text{ all} \cdots]\!]\}$
d. $C^{Dom} = \{\exists, \forall\}$
e. $[\![\text{Exh}]\!](C^{Dom})(Dom([\![\text{only } \text{J}_\text{F} \cdots \text{ some} \cdots]\!]))$
$= [\![\text{Exh}]\!](\{\exists, \forall\})(\exists) = \exists \;\&\; \neg\forall$

In (39), where *some* appears as *only*'s focus associate, we run into a problem. The prejacent presupposition of the sentence is the same as the one in (38), and we have no reason to dismiss the possibility that the sentence has alternatives in which *some* is replaced with a stronger term.[14] We therefore have all the necessary ingredients to enrich (39)'s presupposition in the

[13]In Alxatib 2020 I discuss more thoroughly the applicability and consequences of Spector and Sudo's account to our data, and Marty and Romoli's.

[14]In this case, replacing *some* with *all* gives us an odd sentence: *J only did all$_F$ of the homework*. This does not affect the seriousness of the point, however, since other examples can be constructed where both the weaker term and the stronger term are compatible with *only*: e.g. [*J only used a teaspoon$_F$ of sugar*] and [*J only used a tablespoon$_F$ of sugar*].

same way as in (38). Here, however, we want the negation of *some*'s alternatives to be asserted by *only*, not presupposed upon exhaustification.

This calls for a revision to (40)/(41). Let us subtract from C all those alternative whose presuppositions are falsified already by ϕ. Let us call the resulting set C_*:

(45) Given a proposition ϕ and set of propositions C,
$C_* := C - \{\psi : \psi \in C \,\&\, Dom(\psi) \cap \phi = \varnothing\}$.

In (37) and (38), repeated below, C_* is the same as C: in both cases the $\forall$ presupposition of the *all*-alternative is compatible with the truth conditions of the sentence.

(37) J is unaware that all of the dessert was eaten.

(38) Only $\mathrm{J_F}$ did some of the homework.

In (39), however, C_* by definition must not have the *all*-alternative, because its presupposition — that J did all of the homework — can't be true given the truth conditions of (39).

(39) J only did $\mathrm{some_F}$ of the homework.

We can now rewrite our definedness condition of Exh and limit it to C_*. This way, we remove the redundancy that we ran into with (39): if ϕ is incompatible with the presupposition of an alternative ψ, that is, if by its truth conditions ϕ entails the negation of ψ's presupposition, then ψ is not used in calculating ϕ's presuppositional implicature.

(46) **Definedness condition on Exh (Final)**:
$Dom([\![\mathrm{Exh}]\!](C)(\phi)) \subseteq [\![\mathrm{Exh}]\!](C_*^{Dom})(Dom(\phi))$

Or alternatively:

(47) $[\![\mathrm{Exh}]\!](C)(\phi) = [\lambda w : [\![\mathrm{Exh}]\!](C_*^{Dom})(Dom(\phi))(w){=}1 \,.\, \cdots]$

Now we have everything we need to derive the FC/distribution presuppositions under *only*. Because disjunct alternatives are not targeted by *only*'s assertion, we can simplify the particle's semantics and remove all reference to includable and excludable alternatives. Here I revert to a semantics based on Krifka 1993 but attributed (by van Rooij and Schulz) to Schwarzschild (1994). The entry presupposes *only*'s prejacent and asserts the negation of its non-weaker alternatives.

(48) **Krifka/Schwarzschild's *only***:
$[\![only]\!](C)(\phi) = [\lambda w : \phi(w){=}1 \,.\, \forall\psi(\phi \not\models \psi \rightarrow \psi(w){=}0)]$

However, we will keep Bar-Lev and Fox's definition of exhaustification, repeated:

(32) **Bar-Lev and Fox's Exh**:
$[\![\mathrm{Exh}]\!](C)(\phi) = [\lambda w \,.\, \boldsymbol{inclusions}_C(\phi)(w){=}1 \,\&\, \boldsymbol{exclusions}_C(\phi)(w){=}1]$

Let us apply these semantics to (8) first, then to (7).

(8) M is only required to eat [cake or ice cream]$_\mathrm{F}$. ($only(\Box(p \vee q)_\mathrm{F})$)

With access only to independent alternatives to the prejacent, *only*'s assertion in (8) will say that M is not required to eat anything other than cake or ice cream. Its presupposition is the disjunctive requirement:

(49) $[\![(8)]\!] = [\lambda w : \Box(p \vee q)(w) = 1\ .\ \neg\Box r_1\ \&\ \neg\Box r_2\ \&\ \cdots]$

Now, (8) itself has formal alternatives like the examples just discussed. In the alternatives, the focused disjunction is replaced with its disjuncts (50a,b), independent alternatives (50c), and the conjunction (50d).

(50) Given $\phi = only(\Box(p \vee q)_\mathrm{F})$,
$C = \cdots$
a. $\{only(\Box(p)_\mathrm{F})$,
b. $only(\Box(q)_\mathrm{F})$,
c. $only(\Box(r_1)_\mathrm{F}),\ only(\Box(r_2)_\mathrm{F}),\ \cdots$
d. $only(\Box(p \wedge q)_\mathrm{F})\}$

The alternatives in (50c) have presuppositions that are falsified by the meaning of (8). Therefore they are absent from C_*. The alternatives in (50a,b,d), however, remain.

(51) Given $\phi = only(\Box(p \vee q)_\mathrm{F})$,
$C_* = \{only(\Box(p)_\mathrm{F}),\ only(\Box(q)_\mathrm{F}),\ only(\Box(p \wedge q)_\mathrm{F})\}$

So when the presupposition of (8) is strengthened, it is strengthened relative to the presuppositions of the elements of C_* above. Specifically, by the recipe in (46), and assuming ϕ, C from (50) and the resulting C_* in (51), we get the following (target) result:

(52)
$$\begin{aligned} Dom([\![\mathrm{Exh}]\!](C)(\phi)) &\subseteq [\![\mathrm{Exh}]\!](C_*^{Dom})(Dom(\phi)) \\ &\subseteq [\![\mathrm{Exh}]\!](\{\Box p, \Box q, \Box(p \wedge q)\})(\Box(p \vee q)) \\ &\subseteq \Box(p \vee q)\ \&\ \neg\Box p\ \&\ \neg\Box q \end{aligned}$$

The same steps lead to the FC presupposition in the case of (7), but here it is the inclusions of Exh rather than its exclusions that produce FC.

(7) M is only allowed to eat [cake or ice cream]$_\mathrm{F}$. ($only(\Diamond(p \vee q)_\mathrm{F})$)

(53) $[\![(7)]\!] = [\lambda w : \Diamond(p \vee q)(w) = 1\ .\ \neg\Diamond r_1\ \&\ \neg\Diamond r_2\ \&\ \cdots]$

(54) Given $\phi = only(\Diamond(p \vee q)_\mathrm{F})$,
$C = \cdots$
a. $\{only(\Diamond(p)_\mathrm{F})$,
b. $only(\Diamond(q)_\mathrm{F})$,

c. $only(\Diamond(r_1)_F)$, $only(\Diamond(r_2)_F)$, $\cdots$

d. $only(\Diamond(p \wedge q)_F)\}$

(55) Given $\phi = only(\Diamond(p \vee q)_F)$,
$C_* = \{only(\Diamond(p)_F),\ only(\Diamond(q)_F),\ only(\Diamond(p \wedge q)_F)\}$

(56)
$$\begin{aligned} Dom(\llbracket \text{Exh} \rrbracket(C)(\phi)) &\subseteq \llbracket \text{Exh} \rrbracket(C_*^{Dom})(Dom(\phi)) \\ &\subseteq \llbracket \text{Exh} \rrbracket(\{\Diamond p, \Diamond q, \Diamond(p \wedge q)\})(\Diamond(p \vee q)) \\ &\subseteq \Diamond p \mathbin{\&} \Diamond q \mathbin{\&} \neg\Diamond(p \wedge q) \end{aligned}$$

This concludes the technical discussion.

Notice that the proposal comes with a prediction: both the distribution presupposition and the FC presupposition should be licensed by disjunctive phrases that appear outside of *only*'s focus, much like the presuppositional strengthening seen earlier of unfocused *some*. The prediction is confirmed by the following examples (discussed in Alxatib 2020):

(57) a. Only J_F is allowed to eat cake or ice cream.

b. Is only J_F allowed to eat cake or ice cream?

(58) a. Only J_F is required to eat cake or ice cream.

b. Is only J_F required to eat cake or ice cream?

(57a) can be understood to say that J is free to choose between cake and ice cream, while other people do not have permission to eat either. Similarly, (58a) can be used to say that J is required to eat one (but neither specifically) of the two desserts, while the others are not required to eat either. The status of FC/distribution as a presupposition is shown in the questions in (57b)/(58b).

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