Only, *or*, and free choice presuppositions*

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Abstract

Bar-Lev and Fox (2020), B-L&F, redefine the exhaustification operator, Exh, so that it negates innocently excludable (IE) alternatives and asserts innocently includable (II) ones. They similarly redefine the exclusive particle *only* so that it negates IE-alternatives, and *presupposes* II ones. B&F justify their revision of *only* on the basis of Alxatib's (2014) finding that *only* presupposes free choice (FC) in cases like *Kim was only allowed to eat soup or salad*. I show challenges to B&F's view of *only* and argue against extending II to its meaning. Instead I propose that FC is better treated as a "presuppositional implicature" in such cases. I show the details of how this can be done and identify the necessary (and occasionally novel) auxiliary assumptions.

1 Introduction

In recent work, Bar-Lev and Fox (2020)[1] — hereafter B-L&F — proposed new definitions of the exclusive particle *only* and the exhaustivity operator Exh. On their revisions, the two operators generate inferences not only from what Fox (2007) dubbed the innocently excludable alternatives to the prejacent — the propositional argument to *only*/Exh — but also from what B-L&F call its innocently *in*cludable (II) alternatives. In the case of Exh, the motivation for involving II-alternatives comes from a group of sentences that intuitively license free choice (or free choice-like) inferences — FC — but that are not predicted to do so by earlier, Exh-based accounts of FC. In the case of *only*, which is my main concern, B-L&F's revision is intended to explain why sentences like (1) *presuppose* FC (Alxatib 2014).

(1) Kim is only allowed to eat [soup or salad]$_{\mathrm{F}}$.
Presupposition: Kim is allowed to eat soup, Kim is allowed to eat salad.

In what follows, I will argue that B-L&F's account of *only*, and of (1), is incorrect. In my argument, I will cite other cases where *only* seems to presuppose FC, but where appeal to Innocent Inclusion does not help. These cases show that FC presuppositions under *only*

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[1] Also Bar-Lev and Fox (2017) and Bar-Lev (2018).

have another source, which leaves B-L&F's recruitment of II-alternatives unmotivated. I will present an account of *only*'s FC presuppositions as "presuppositional implicatures", following the proposals of Gajewski and Sharvit (2012) and Marty and Romoli (2020).

2 Background

A reasonable hypothesis about the semantics of *only* is that it (a) presupposes its prejacent, and (b) asserts that every alternative that does not follow from it is false. For example, (2) presupposes that Kim ate soup, and asserts that s/he did not eat other things:

(2) Kim only ate [soup]$_{\mathrm{F}}$.

  a. *Presupposition*: Kim ate soup.

  b. *Assertion*: Kim did not eat salad, Kim did not eat rice, ...

We may therefore write, following Krifka (1993), that given a proposition ϕ, set of "alternative" propositions C, and world of evaluation w,

(3) $\phi \in Dom([\![only]\!]^w(C))$ only if $\phi(w)=1$; if $\phi \in Dom([\![only]\!]^w(C))$, then $[\![only]\!]^w(C)(\phi)=1$ iff $\forall\psi(\psi \in C \,\&\, \phi \not\subseteq \psi \rightarrow \psi(w)=0)$

Here and in the rest of the paper, I will treat presuppositions as definedness conditions on propositions (as in e.g. Heim and Kratzer 1998). A proposition ϕ that carries a contingent presupposition is a partial function from possible worlds to truth values. Its (partial) domain, $Dom(\phi)$, is the set of worlds where it is defined, i.e. its presupposition. As we move on, I will drop some of the notational clutter to increase readability, but for the time being I will build on the formal presumptions in (3).

As is known, there is an issue with the characterization in (3). If *only* takes a disjunctive focus associate, as in (4), and if we follow Sauerland (2004) and assume that disjunctions have their disjuncts as formal alternatives, then the presupposition of *only* is predicted to contradict its assertion ((4c,d) below). Sentences of this form are not contradictory, however.

(4) Kim only ate [soup or salad]$_{\mathrm{F}}$.

  a. (4) $\approx only_C(p \vee q)$

  b. $C=\{p,q,r,p\wedge q\}$

  c. $[\![(4)]\!]^w$ is defined only if $[p\vee q](w)=1$

  d. If defined, $[\![(4)]\!]^w=1$ iff $p(w)=0$ and $q(w)=0$ (and $r(w)=[p\wedge q](w)=0$)

To prevent this outcome, we follow Fox (2007) and change the condition on excludability. We say that a proposition ψ is *innocently* excludable (IE), given a prejacent proposition ϕ and set of alternatives C, iff $\neg\psi$ does not affect the consistency of any selection of exclusions from C. By "selection of exclusions from C", we mean the conjunction of the negations of a subset of C. For example, let $\phi=p\vee q$, and $C=\{p,q,r,p\wedge q\}$, as on the current example. Is p innocently excludable? The answer is no. To see why, take this subset of C: $B=\{q\}$. The selection of exclusions associated with B is the proposition $\neg q$. This proposition is consistent with ϕ: $(p\vee q)\,\&\,\neg q$ is not contradictory. However, adding

$\neg p$ makes this set of exclusions inconsistent. Therefore, there is a selection of exclusions from C (given $p \vee q$) whose consistency is affected by $\neg p$. For this reason, p is not IE. The same logic makes q non-IE, leaving r and $p \wedge q$ as IE.

Let us then write that *only* presupposes its prejacent and asserts that every IE-alternative of it is false (see (5)). We define $\mathrm{IE}(C)(\phi)$ as that subset of C that contains ψ iff ψ satisfies the condition described above, namely: for every subset B of C, if the negations of B's elements are jointly consistent with ϕ, then so are the negations of B's elements together with $\neg\psi$ (see (6)).[2]

(5) $\phi \in Dom([\![only]\!]^w(C))$ only if $\phi(w)=1$; if $\phi \in Dom([\![only]\!]^w(C))$, then $[\![only]\!]^w(C)(\phi)=1$ iff $\forall\psi(\psi \in \mathrm{IE}(C)(\phi) \rightarrow \psi(w)=0)$

(6) $\mathrm{IE}(C)(\phi) = \{\psi : \psi \in C \;\&\; \forall B(B \subseteq C \;\&\; (B^{\neg} \wedge \phi \nvDash \bot) \rightarrow (B^{\neg} \wedge \phi \wedge \neg\psi \nvDash \bot))\}$

From (5), (4) is no longer predicted to be contradictory. It is now predicted to presuppose that Kim ate soup or salad, and assert that Kim did not eat both, and did not eat anything else.[3]

In what follows, I will write $IE^{\neg}_{\phi,C}$ when I mean the conjunction of the negations of ϕ's IE-alternatives in C (7).

(7) $IE^{\neg}_{\phi,C} = [\lambda w \,.\, \forall\psi(\psi \in \mathrm{IE}(C)(\phi) \rightarrow \psi(w)=0)]$

With that, we may rewrite (5) as in (5′):

(5′) $\phi \in Dom([\![only]\!]^w(C))$ only if $\phi(w)=1$; if $\phi \in Dom([\![only]\!]^w(C))$, then $[\![only]\!]^w(C)(\phi)=1$ iff $IE^{\neg}_{\phi,C}(w)=1$

The motivation for working IE into the semantics of *only*, as outlined above, applies also to Exh, the silent operator that some believe to generate scalar implicatures (SIs). Unlike *only*, Exh *asserts* its prejacent, but otherwise the two operators are alike in negating their prejacents' alternatives. When Exh's prejacent is a disjunction, Exh must be kept from negating the individual disjuncts if contradictions are to be avoided. Exh's exclusions are therefore limited to $\mathrm{IE}(C)(p)$, given a prejacent p and set of alternatives C:

(8) $[\![\mathrm{Exh}]\!]^w(C)(\phi)=1$ iff $\phi(w)=1 \;\&\; IE^{\neg}_{\phi,C}(w)=1$ (to be revised)

Exhaustifying a plain disjunction $p \vee q$ therefore entails the exclusive inference $\neg(p \wedge q)$, and the negations of other alternatives r:

(9) $[\![\mathrm{Exh}]\!]^w(C)(p \vee q)=1$ iff $[p \vee q](w)=1 \;\&\; [p \wedge q](w)=0 \;\&\; r(w)=0$

[2]Notice that any alternative ψ that is entailed by ϕ is predicted by this definition to not be IE with respect to it. This is because the exclusions that come from the empty set, which is a subset of C, are consistent with ϕ, but when $\neg\psi$ is added the result is contradictory. Later I will show that, when presuppositions are considered, this definition of IE makes the same prediction for alternatives that are Strawson-entailed by ϕ.

[3]I should say that I do not think that *only* operates on the conjunctive alternative to a disjunctive associate. For example, it is awkward to answer the question *Did Kim only eat [soup or salad]*$_{\mathrm{F}}$*?* with *No, s/he ate both* — certainly less awkward than answering it with *No, she also ate rice*. This may indicate that *only* does not "see" the *and*-alternative when its associate is a disjunctive phrase, but I will neither assume nor investigate this here, as it has little effect on my main points. In Section 4 I will make a similar claim, but that claim will have important consequences: I will propose that *only* does not see the individual disjuncts (p,q) as alternatives to a disjunctive associate.

2.1 Free choice inferences and innocent inclusion

Disjunctions present more theoretical challenges. Sentences like (10), as is well known, intuitively license what appears to be a conjunctive inference (separated into (10a,b)). This is the free choice inference (FC).

(10) Kim is allowed to eat soup or salad. $(\Diamond(p \vee q))$
 a. $\rightsquigarrow$ Kim is allowed to eat soup. $(\Diamond p)$
 b. $\rightsquigarrow$ Kim is allowed to eat salad. $(\Diamond q)$

Following insights from Kratzer and Shimoyama 2002 and Alonso-Ovalle 2005, Fox (2007) proposed to derive FC as an implicature by applying Exh recursively (not shown). B-L&F also derive FC as an implicature, but do so from single application of an enriched Exh. Before we see the enrichment, notice that applying the current Exh singly does not generate FC. If $\phi = \Diamond(p \vee q)$, and $C = \{\Diamond p, \Diamond q, \Diamond r, \Diamond(p \wedge q)\}$, then neither $\Diamond p$ nor $\Diamond q$ is IE, for the same reason why neither p nor q are IE in the previous case. Therefore:

(11) Given $C = \{\Diamond p, \Diamond q, \Diamond r, \Diamond(p \wedge q)\}$,
 a. $\mathrm{IE}(C)(\Diamond(p \vee q)) = \{\Diamond r, \Diamond(p \wedge q)\}$
 b. $[\![\mathrm{Exh}]\!]^w(C)(\Diamond(p \vee q)) = [\Diamond(p \vee q)](w) = 1 \ \& \ [\Diamond r](w) = 0 \ \& \ [\Diamond(p \wedge q)](w) = 0$

B-L&F's enrichment of Exh consists of adding to (8) the *assertions* of ϕ's innocently *in*cludable alternatives. A proposition ψ is innocently includable (II), given prejacent ϕ and set of alternatives C, iff ψ does not affect the consistency of any selection of *in*clusions from C. A "selection of inclusions from C" is the grand conjunction of a subset of C. Such a selection is *consistent* (given ϕ, C) iff it is consistent with ϕ along with $IE^{\neg}_{\phi,C}$. The update to the definition of Exh is shown in (12). Innocent Includability is defined in (14).

(12) $[\![\mathrm{Exh}]\!]^w(C)(\phi) = 1$ iff $\phi(w) = 1$ & (prejacent asserted)
 $IE^{\neg}_{\phi,C}(w) = 1$ & (IE-alternatives negated)
 $II_{\phi,C}(w) = 1$ (II-alternatives asserted)

(13) $II_{\phi,C} = [\lambda w \,.\, \forall \psi(\psi \in \mathrm{II}(C)(\phi) \rightarrow \psi(w) = 1)]$

(14) $\mathrm{II}(C)(\phi) = \{\psi : \psi \in C \ \&$
 $\forall B(B \subseteq C \ \& \ (\bigwedge B \wedge \phi \wedge IE^{\neg}_{\phi,C} \nvDash \bot) \rightarrow (\bigwedge B \wedge \phi \wedge IE^{\neg}_{\phi,C} \wedge \psi \nvDash \bot))\}$

Let us illustrate with examples. With plain disjunction, we find that the enrichment makes no difference. Let $\phi = [p \vee q]$ and $C = \{p, q, r, p \wedge q\}$. Is p innocently includable? The answer once again is no. Take this subset of C: $B = \{q\}$. The selection of inclusions associated with B — its grand conjunction, that is — is q. On its own, the inclusion is consistent with $\phi \wedge IE^{\neg}_{\phi,C}$; the proposition $((p \vee q) \wedge \neg(p \wedge q) \wedge \neg r) \wedge q$ is not contradictory. However, adding p to this produces a contradiction, owing to the exclusive inference. Therefore there is a selection of inclusions from C (given $[p \vee q], C$) whose consistency is affected by p. For this reason, p is not II. The same logic makes q non-II. Notice that neither r nor $p \wedge q$ is II, because asserting either one of them directly contradicts $IE^{\neg}_{\phi,C}$.[4]

[4] In more technical terms, asserting r or $p \wedge q$ affects the consistency of the inclusions that come from $\{\}$: $\{\}$ is a subset of C, and its grand conjunction is trivially consistent with $(\phi \wedge IE^{\neg}_{\phi,C})$. Adding r or $p \wedge q$ produces a contradiction because they are inconsistent with $IE^{\neg}_{\phi,C}$. See also footnote 2.

With permission disjunctions like (10), however, the revised definition of Exh in (12) does make a difference. Let $\phi = \Diamond(p \vee q)$ and $C = \{\Diamond p, \Diamond p, \Diamond r, \Diamond(p \wedge q)\}$. Then, as we saw above, $\phi \,\&\, IE^{\neg}_{\phi,C}$ is the proposition $\Diamond(p \vee q) \,\&\, \neg\Diamond r \,\&\, \neg\Diamond(p \wedge q)$. Adding both $\Diamond p$ and $\Diamond q$ to this proposition is consistent, so both alternatives are II.[5] We now get FC:

(15) Given $C = \{\Diamond p, \Diamond q, \Diamond r, \Diamond(p \wedge q)\}$,

$$\begin{aligned} [\![\text{Exh}]\!]^{w}(C)(\Diamond(p \vee q)) = 1 \text{ iff } & [\Diamond(p \vee q)](w) = 1 \;\& && \text{(prejacent)} \\ & [\Diamond r](w) = 0 \;\&\; [\Diamond(p \wedge q)](w) = 0 && (IE^{\neg}_{\phi,C}) \\ & [\Diamond p](w) = 1 \;\&\; [\Diamond q](w) = 1 && (II_{\phi,C}) \end{aligned}$$

As I said, B-L&F motivate their revision of Exh with empirical points beyond $\Diamond(p \vee q)$. These include: cases like (16), where the permission disjunction appears in the scope of a universal quantifier; related cases like (17), where the quantifier *every* is replaced with its contrary *no*, and permission and disjunction are replaced with necessity and conjunction, respectively; cases like (18), where a universal quantifier separates the permission modal from the disjunction; and cases of SDA (Simplification of Disjunctive Antecedents) like (19), where a conditional containing a disjunctive antecedent licenses inferences to conditionals like it, but where the disjunction is replaced with the disjuncts.[6] Each of these licenses an FC-like inference, but none of them can be captured on the original definition of Exh.[7] The inferences are shown below, but in the interest of brevity I do not review B-L&F's II-based account of them.

(16) Everyone is allowed to eat soup or salad. $(\forall(\Diamond(p \vee q)))$
   a. $\rightsquigarrow$ Everyone is allowed to eat soup. $(\forall\Diamond p)$
   b. $\rightsquigarrow$ Everyone is allowed to eat salad. $(\forall\Diamond p)$

(17) No-one needs to eat soup and salad. $(\neg\exists\Box(p \wedge q))$
   a. $\rightsquigarrow$ No-one needs to eat soup. $(\neg\exists\Box p)$
   b. $\rightsquigarrow$ No-one needs to eat salad. $(\neg\exists\Box q)$

(18) Kim is okay with everyone eating soup or salad. $(\Diamond\forall(p \vee q))$
   a. $\rightsquigarrow$ Kim is okay with everyone eating soup. $(\Diamond\forall p)$
   b. $\rightsquigarrow$ Kim is okay with everyone eating salad. $(\Diamond\forall q)$

(19) If Kim eats soup or salad, s/he will be healthy. $((p \vee q) \,\Box\!\!\rightarrow r)$
   a. $\rightsquigarrow$ If Kim eats soup, s/he will be healthy. $(p \,\Box\!\!\rightarrow r)$
   b. $\rightsquigarrow$ If Kim eats salad, s/he will be healthy. $(q \,\Box\!\!\rightarrow r)$

[5] As the reader may have noticed, if we assume that ϕ is a formal alternative to itself, then it too will be innocently includable. Strictly speaking, then, there is no need write into Exh the assertion of the prejacent, because that follows from $II_{\phi,C}$ anyway. For clarity I will still write the assertion of the prejacent separately throughout the paper.

[6] Examples (16) and (17) are based on Chemla 2009; (18) is inspired by data from Nouwen 2018, and (19) is built on an old debate concerning counterfactual conditionals and the inferences that they license (Stalnaker 1968, Lewis 1973, Fine 1975, Nute 1975).

[7] The noted inferences in (16) can be derived if Exh appears recursively in the scope of *everyone*. However, B-L&F argue that a global derivation of the inference must be available also.

As B-L&F note, making use of II-alternatives in the definition of Exh constitutes a further departure from the (neo-)Gricean view of implicature generation. They do not see the departure to be without conceptual appeal, however, since the revision assigns to Exh the role of *settling*, rather than negating, as many formal alternatives to the prejacent as consistency allows. Exh's function can therefore be seen to determine, as narrowly as possible, the cell in the partition of logical space that its propositional argument identifies. The more information there is about the truth values of the alternatives, the narrower the cell.[8]

3 Innocent Inclusion and *only*

Before we return to FC and *only*, it is worth remarking that any conceptual points of support for rewriting Exh should not be thought by default to carry over to *only*. Indeed, differences between *only* and exhaustification have been pointed out and discussed in the literature (see e.g. Bonomi and Casalegno 1993, Alxatib 2013, Buccola 2018). So, to the extent that we are willing to grant that Exh serves the function sketched above, little follows about whether *only* serves a similar function. My more specific point is that, whatever role we think is played by II-alternatives in the definition of Exh, that role need not be the same in the case of *only*, nor does it need to have an analog there in the first place. We may therefore accept B-L&F's revision of Exh without committing to such a revision in the case of *only*. This is the position that I will take.

Let us return to (1). FC in this case is presupposed, not asserted.

(1) Kim is only allowed to eat [soup or salad]$_{\mathrm{F}}$.
Presupposition: Kim is allowed to eat soup, Kim is allowed to eat salad.

B-L&F propose to derive this result by having *only* presuppose II-alternatives. Thus the inferences that are licensed by Exh are, in their content, the same as those licensed by *only*; the difference is that, with *only*, the prejacent's II-alternatives are presupposed (see (20a)), while with Exh they are asserted along with the negations of the IE-alternatives:

(20) Given a proposition ϕ, set of propositions C, and world of evaluation w,
a. $\phi \in Dom([\![only]\!]^w(C))$ only if $II_{\phi,C}(w) = 1$ (II-alts are presupposed)
b. If $\phi \in Dom([\![only]\!]^w(C))$, then
$[\![only]\!]^w(C)(\phi) = IE^{\neg}_{\phi,C}(w)$ (IE-alts are negated)

Given that $\Diamond p, \Diamond q$ are innocently includable with respect to $\phi = \Diamond(p \vee q)$ and its (by now familiar) alternatives, we get the FC presupposition in the case of (1), as desired:

(21) Given $C = \{\Diamond p, \Diamond q, \Diamond r, \Diamond(p \wedge q)\}$,
a. $\Diamond(p \vee q) \in Dom([\![only]\!]^w(C))$ only if $[\Diamond p](w) = 1$ &
$[\Diamond q](w) = 1$ (II presupposed: FC!)
b. If $\Diamond(p \vee q) \in Dom([\![only]\!]^w(C))$, then
$[\![only]\!]^w(C)(\Diamond(p \vee q)) = 1$ iff $[\Diamond r](w) = 0$ &
$[\Diamond(p \wedge q)](w) = 0$ (IE)

[8]See Bar-Lev and Fox 2020 (Sect. 3) for discussion of this point, and also Fox 2018 for applications to question semantics.

Later I will propose a different take on (1), where the FC presupposition is derived not from a revised entry for *only*, but from applying exhaustification externally to it (as suggested in Alxatib 2014, building on Gajewski and Sharvit 2012, and proposed for other cases in Marty and Romoli 2020). I will compare that proposal to B-L&F's by looking at three other cases of FC presupposition under *only*.

3.1 Case 1: *only*$(\Box(p\vee q))$

Consider (22).

(22) Kim only needs to eat [soup or salad]$_{\mathrm{F}}$.
Presupposition: Kim does not need to eat soup, Kim does not need to eat salad.

Here *only*'s associate is a disjunction, as in (1), but the modal that appears in the prejacent is universal. The sentence presupposes its prejacent, as expected, but it also seems to presuppose an additional inference of free choice: Kim is not required to eat soup and not required to eat salad. The inference is intuitively licensed by the interrogative version of the sentence, shown in (23).

(23) Does Kim only need to eat soup or salad?
Presupposition: Kim does not need to eat soup, Kim does not need to eat salad.

Example (22) presents a similar question to (1), namely, why it is that *only*'s presupposition contains more than what appears to be the literal content of its prejacent. In the case of (1), B-L&F's answer was that *only* presupposes all of its prejacent's II-alternatives. In (22), however, this move does not help. I expand on this in what follows.

On the view of alternatives that we have assumed so far, the prejacent in (22), which has the form $\Box(p\vee q)$, has other necessity claims as alternatives: $\Box p$, $\Box q$, $\Box r$, and $\Box(p\wedge q)$. These alternatives are all innocently *ex*cludable, for the simple reason that their negations are jointly consistent with the prejacent.

(24) Given $C=\{\Box p,\Box q,\Box r,\Box(p\wedge q)\}$,

a. $\Box(p\vee q)\in Dom([\![only]\!]^w(C))$ only if $[\Box(p\vee q)](w)=1$ (II: *prejacent only*)

b. If $\Box(p\vee q)\in Dom([\![only]\!]^w(C))$, then
$[\![only]\!]^w(C)(\Box(p\vee q))=1$ iff $[\Box p](w)=0$ &
$[\Box q](w)=0$ &
$[\Box r](w)=0$ &
$[\Box(p\wedge q)](w)=0$ (IE)

So the problem from (22) is, in a sense, two-sided: the desired inference ($\neg\Box p,\neg\Box q$) is intuitively presupposed, but is not predicted to be; instead it is predicted to be asserted. It must be remarked here that this prediction holds on B-L&F's view as it does on the standard view of *only*: as long as the alternatives to the prejacent are assumed to be those in C in (24), we predict the FC-like inference in this case to be part of *only*'s assertion. In Section 4 I will propose that *only* does not operate on alternatives to a prejacent that result from replacing a focused disjunction with its disjuncts. This will remove the negations of $\Box p, \Box q$ from its assertion.[9]

What ways are there around the outcome in (24)? If we work within B-L&F's view, we can try to add to C the alternatives that result from replacing the necessity modal with the possibility modal. The relevant additions are $\Diamond p$ and $\Diamond q$:

(25) $C = \{\Box p, \Box q, \Box r, \Box(p \wedge q), \Diamond p, \Diamond q, \Diamond r, \Diamond(p \wedge q)\}$

This correctly moves the inference $(\neg\Box p, \neg\Box q)$ away from *only*'s assertion, but unfortunately it does not move it to *only*'s presupposition, because none of the elements of C are II. Let me explain the first prediction first. From the expansion of C in (25), $\Box p, \Box q$ are no longer predicted to be IE, because of the alternatives $\Diamond q, \Diamond p$, respectively: $\{\Diamond q\}$ is a subset of C, and the negation of its element is consistent with the prejacent $\Box(p \vee q)$, but adding the negation of $\Box p$ leads to a contradiction. Therefore the negation of $\Box p$ affects the consistency of a subselection of exclusions from C, namely $\{\Diamond q\}^{\neg}$, and is therefore not IE. By the same logic, $\Box q$ is not IE, this time because of $\Diamond p$. So far this takes the negations of $\Box p, \Box q$ from the assertion of *only*, as desired:

(26) Given $C = \{\Box p, \Box q, \Box r, \Box(p \wedge q), \Diamond p, \Diamond q, \Diamond r, \Diamond(p \wedge q)\}$,
   a. $\mathrm{IE}(C)(\Box(p \vee q)) = \{\cancel{\Box p}, \cancel{\Box q}, \Box r, \Box(p \wedge q), \cancel{\Diamond p}, \cancel{\Diamond q}, \Diamond r, \Diamond(p \wedge q)\}$
   b. $IE^{\neg}_{\Box(p \vee q), C} = [\lambda w \,.\, [\Diamond r](w) = 0 \;\&\; [\Diamond(p \wedge q)](w) = 0]$

(27) a. $\Box(p \vee q) \in Dom([\![only]\!]^{w}(C))$ only if ... (No FC yet; see below)
   b. If $\Box(p \vee q) \in Dom([\![only]\!]^{w}(C))$, then
   $[\![only]\!]^{w}(C)(\Box(p \vee q)) = 1$ iff $[\Diamond r](w) = 0$ &
   $[\Diamond(p \wedge q)](w) = 0$ (IE)

What about the II-alternatives? Clearly $\Box p, \Box q$ are not simultaneously consistent with the IE-exclusions in (26b) — individually they are, but not together. Therefore, neither is II, and neither will be presupposed by *only*. So far, this is good. But the same thing holds of the pair $\Diamond p, \Box q$. These are also individually consistent with the IE-exclusions derived above, but together they lead to a contradiction: if $(p \vee q)$ is required and $(p \wedge q)$ is prohibited, then it is possible for q to be required — asserting $\Box q$ is consistent so far — but then p can't be permitted, because that would make $(p \wedge q)$ permitted too, contrary to the inference that it is prohibited. It follows that neither $\Diamond p$ nor $\Box q$ is II, and by the same reasoning, that neither $\Diamond q$ nor $\Box p$ is II.

So, while we have successfully kept $\Box p, \Box q$ from being IE, and therefore out of the reach of *only*'s assertion, we have not yet managed to add any inferences to its presuppositional component. We can reach that goal with a more stipulative move, but as we will see, the results will still not be quite what we want. Suppose we remove $\Diamond(p \wedge q)$ from C, but keep $\Box(p \wedge q)$:

(28) $C = \{\Box p, \Box q, \Box r, \Box(p \wedge q), \Diamond p, \Diamond q, \Diamond r, \cancel{\Diamond(p \wedge q)}\}$

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[9] Crucially, note that this position is not available to B-L&F, because it would (in parallel) remove $\Diamond p$ and $\Diamond q$ from C in the case of (1), and would therefore leave no II-alternatives for *only* to presuppose.

Then *only*'s assertion will be weakened (see (30b)), because now the only exclusion that concerns the conjunction $(p \wedge q)$ is that it is not required:

(29) Given $C = \{\Box p, \Box q, \Box r, \Box(p\wedge q), \Diamond p, \Diamond q, \Diamond r\}$,
$\text{IE}(C)(\Box(p\vee q)) = \{\cancel{\Box p}, \cancel{\Box q}, \Box r, \Box(p\wedge q), \cancel{\Diamond p}, \cancel{\Diamond q}, \Diamond r\}$

(30) a. $\Box(p\vee q) \in Dom([\![only]\!]^w(C))$ only if $[\Diamond p](w)=1$ & $[\Diamond q](w)=1 \ldots$ (II – FC)

b. If $\Box(p\vee q) \in Dom([\![only]\!]^w(C))$, then
$[\![only]\!]^w(C)(\Box(p\vee q))=1$ iff $[\Diamond r](w)=0$
$[\Box(p\wedge q)](w)=0$ (IE)

And because we do not have the alternative $\Diamond(p\wedge q)$, we will admit $\Diamond p, \Diamond q$ into the set of II-alternatives: each is consistent with any (consistent) set of assertions, given the exclusions in (30b). (I leave it to the reader to verify this.) With $\Diamond p, \Diamond q$ as II-alternatives, we predict (on first glance correctly) that *only* presuppose $\Diamond p, \Diamond q$ in the case of *only*$(\Box(p\vee q)_{\text{F}})$.

This result has several problems, however. First, the alternatives that were generated in (25) and (28) require modification of material outside of *only*'s focus associate. To show that this is problematic, it must be shown (i) that alternatives to *only* cannot vary by more than its focus associate, and (ii) that the associate in (22) does not include the modal. Point (i) is easily demonstrated with the difference in meaning between (31a) and (31b). Indeed the literature is replete with examples like these, precisely to show the role of the associate in generating alternatives:

(31) a. Kim only washed$_{\text{F}}$ the cilantro.
Paraphrase: 'Kim did not do anything to the cilantro other than wash it.'

b. Kim only washed the cilantro$_{\text{F}}$.
Paraphrase: 'Kim did not wash anything other than the cilantro.'

Both (31a,b) presuppose that Kim washed the cilantro. But as indicated above, the sentences make different assertions depending on where the focus is understood to be (accenting makes that clear in these examples). If the alternatives to the prejacent could vary by more than the focus-marked element, then (31a,b) would make identical assertions, because in (31a) the alternatives would differ by the focused verb as well as the unfocused object, and likewise, *mutatis mutandis*, in (31b).

Point (ii) is shown by the following modification of the example:

(32) What does Kim need to eat? Does s/he only need to eat soup or salad?[10]

Example (32) has two questions, a *wh*-question and a yes/no-question. The yes/no-question expresses a guess about the answer to the *wh*-question. My intention here is to use the necessary congruence between the two questions to determine more clearly the focus-associate of *only* in the yes/no-question. In doing this, I am assuming Rooth's (1992) Question-Answer Constraint, which says that a focused answer (here in association with *only*) to a *wh*-question is appropriate only if the prejacent's alternatives in the answer form a subset

[10]With the right intonation, (32) can also be read as an alternative question. This is not the intended reading here.

of the question's denotation (Hamblin 1973). By Rooth's theory of focus semantics, alternatives are generated by replacements to the focus-marked element (*only*'s associate in this case). If this holds, and if the denotation of the *wh*-question consists of propositions about Kim's eating requirements, then the focus in the second question must be limited to the disjunctive phrase [*soup or salad*] — otherwise the alternatives would not form a subset of the denotation of the *wh*-question. Importantly, the inference that Kim does not need to eat soup and does not need to eat salad is still licensed in (32).[11,12]

The second problem with the result is that it relies on a non-uniform account of alternative generation: we need to have $\Box(p \wedge q)$ as an alternative to $\Box(p \vee q)$, because otherwise $\Box p$ and $\Box q$ would be II and would lead, incorrectly, to the presupposition that both p, q are required. At the same time, we need $\Diamond(p \wedge q)$ to not be an alternative, because keeping it prevents $\Diamond p, \Diamond q$ from being II, and hence from being presupposed. I do not know of a principled account of alternatives that can accommodate these assumptions.

The third problem is that, even with these unwanted assumptions, the inferences we derived are still too weak. From the view of alternatives in (29), the result consisted of (a) the presupposition that $\Box(p \vee q)$ & $\Diamond p$ & $\Diamond q$, and (b) the assertion that $\neg\Diamond r$ & $\neg\Box(p \wedge q)$, etc. These were shown in (30), repeated here:

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[11] This may look like a roundabout demonstration: why did we not use question-answer congruence with a *wh*-question and a declarative answer, as in (i)?

(i) Q. What does Kim need to eat?
A. S/he only needs to eat soup or salad.

The reason is that there may be a confound. I want to show that FC is presupposed in these cases, and also that the disjunctive phrase alone is *only*'s associate. In (i) the answer does not intuitively *presuppose* FC, nor that Kim needs to eat soup or salad — indeed, how can either inference be presupposed if they are part of an informative answer to the question? This is a more general issue with *only*, however: independently of modality and FC, *only*'s prejacent "presupposition" is known to be usable as an answer to a question, as in (ii):

(ii) Q. What did Kim eat?
A. S/he only ate soup.

To control for this confound and highlight the presuppositional status of the prejacent, we can turn the answer into a follow-up "guess", in the form of a yes/no-question. Doing this does indeed show *only*'s prejacent to be presuppositional. Note the difference between (a) and (b) below:

(iii) a. What did Kim eat? Did she eat soup? (Without *only*, no presupposition about soup)
b. What did Kim eat? Did she only eat soup? (With *only*, presupposition that Kim ate soup)

[12] The same point is shown by the simpler but somewhat different (i):

(i) The only thing that Kim needs to eat is soup or salad.

Example (i) licenses the same inferences as (22): Kim has a requirement to eat one of soup/salad; has no requirement to eat either specifically; and has no requirement to eat anything else. And like in (22), the inferences concerning soup/salad in (i) are presupposed, as the interrogative (ii) shows:

(ii) Is the only thing that Kim needs to eat soup or salad?

Again, however, the structure of the sentence shows clearly that its focus is the food — the disjunction [*soup or salad*] — and does not concern the modality.

(30) a. $\Box(p\vee q)\in Dom([\![only]\!]^w(C))$ only if $[\Diamond p](w)=1$, $[\Diamond q](w)=1$ (II)

b. If $\Box(p\vee q)\in Dom([\![only]\!]^w(C))$, then
$[\![only]\!]^w(C)(\Box(p\vee q))=1$ iff $[\Diamond r](w)=0$
$[\Box(p\wedge q)](w)=0$ (IE)

The problem is that these inferences are compatible with a scenario where p is *required* and where q is optional. But in such a scenario, the sentences of interest are infelicitous: if Kim *needs* to eat soup, and can (but does not need to) eat salad, then both the declarative and interrogative forms (33a,b) are misleading:

(33) a. Kim only needs to eat soup or salad. (=(22))

b. Does Kim only need to eat soup or salad? (=(23))

So, to get the FC presupposition in these cases, it is not enough to presuppose the alternatives $\Diamond p, \Diamond q$. What we need is the presupposition that $\neg\Box p$ and $\neg\Box q$, and getting this result does not seem possible on B-L&F's account of *only*.

Finally, setting aside focus-marking and turning instead to exhaustification, there are reasons why the implicatures of $\Box(p\vee q)$ — e.g., *Kim needs to eat soup or salad* — should be calculated without the alternatives $\Diamond p, \Diamond q$. It is clear from the previous discussion that admitting these alternatives blocks the implicatures $\neg\Box p, \neg\Box q$. B-L&F note this, but they do so in the context of the distributive inferences of sentences like (34):

(34) Everyone ate soup or salad.

The logic here is identical to necessity modals.[13] Intuitively, (34) suggests (on first glance at least) that not everyone ate soup and not everyone ate salad. These (distributive) implicatures can be derived if we exhaustify (34) against its disjunct alternatives — *everyone ate soup*, *everyone ate salad* — but not if we also admit the alternatives *someone ate soup* and *someone ate salad*. B-L&F argue that admitting these latter alternatives (and consequently blocking the distributive implicatures) may be good after all, in light of findings from Crnič et al. 2015. However, B-L&F also note (following Crnič et al.) that the situation is different for modals, so the reasons for admitting the *someone*-disjunct alternatives to (34) do not carry over to cases like *Kim needs to eat soup or salad*. Therefore, even without focus-marking to tell us what we can and cannot replace, there are reasons to keep $\Diamond p$ and $\Diamond q$ from being alternatives to $\Box(p\vee q)$.[14]

I conclude that in examples that represent Case 1, i.e. sentences of the form $only\Box(p\vee q)_{\mathrm{F}}$, the formal alternatives to the prejacent do not include $\Diamond p$ and $\Diamond q$. The case therefore remains out of reach for B-L&F.

3.2 Presupposed Ignorance?

There is a theoretical possibility that is compatible with B-L&F's view, and that may seem at first to predict the FC presupposition of $only(\Box(p\vee q)_{\mathrm{F}})$. This is Spector and Sudo's

[13]This is on the standard assumption that necessity modals denote universal quantifiers over possible worlds/situations.

[14]See Crnič et al. 2015 and Bar-Lev and Fox 2020 (Sect. 5.5) for discussion.

(2017) Presupposed Ignorance Principle. The principle blocks uses of a form ϕ if it has an alternative ψ with a stronger, true presupposition:

(35) **Presupposed Ignorance Principle (PIP)**:
If $\psi \in \mathrm{ALT}(\phi)$ and $Dom(\llbracket\psi\rrbracket^{\cent}) \subset Dom(\llbracket\phi\rrbracket^{\cent})$, then $*\phi$ if common ground supports $Dom(\llbracket\psi\rrbracket^{\cent})$.

Spector and Sudo motivate the PIP on the basis of examples like (36).[15]

(36) Kim is unaware that some of our students smoke.

This sentence is odd in contexts where it is known that all of our students smoke, and the PIP correctly predicts this, because in these contexts the stronger presupposition of (37) is supported:

(37) Kim is unaware that all of our students smoke.

We now ask whether the PIP may also help us block $\phi = only(\Box(p \vee q)_{\mathrm{F}})$ in contexts where $\Box p/\Box q$ is true, that is, whether it predicts $\phi = only(\Box(p \vee q)_{\mathrm{F}})$ to imply the falsity of $\Box p/\Box q$. At first the answer appears to be yes, because in such contexts the stronger presuppositions of the alternatives $\psi = only(\Box p_{\mathrm{F}})/only(\Box q_{\mathrm{F}})$ are supported. B-L&F's account may therefore be combined with the PIP to generate all of our target inferences so far: in $only(\Diamond(p \vee q)_{\mathrm{F}})$ the II presupposition of *only* generates FC, and in $only(\Box(p \vee q)_{\mathrm{F}})$ the PIP generates the inferences $\neg\Box p, \neg\Box q$.[16]

But a closer look at the details shows that this isn't at all trivial and takes us back to some of the undesirable assumptions identified above. To see the issue, consider first the simple case of (38):

(38) Kim only used [a teaspoon]$_{\mathrm{F}}$ of vinegar.

The PIP makes a bad prediction here, that (38) is only good in contexts that do not support the (stronger) presupposition of (39):[17]

(39) Kim only used [a tablespoon]$_{\mathrm{F}}$ of vinegar.

That is, if it is known in context that Kim used a tablespoon of vinegar, then (38) should be infelicitous. In fact, the sentence isn't infelicitous in such a context. It is false.

So on the one hand, the PIP puts (36) in competition with (37), and from this competition it generates the correct judgement about (36). But by the same logic, the principle

[15]The discussion in Spector and Sudo 2017 is much more complicated than I make it out to be here, and their motivation of the PIP does not rest solely on examples like (36). My short description of the account is sufficient for my purposes however.

[16]Note that the PIP is not predicted to be problematic in the case of $only\,\Diamond(p \vee q)_{\mathrm{F}}$. By assumption (following B-L&F, that is) the sentence presupposes FC, and FC is *stronger* than the presuppositions of the alternatives $only\,\Diamond p_{\mathrm{F}}$ and $only\,\Diamond p_{\mathrm{F}}$.

[17]In showing this problem it was necessary to use examples that do not involve *some* as a focus associate, because alternatives where *all* takes *some*'s place as associate are odd (e.g., *Kim only ate* $\{$✓*some*$_{\mathrm{F}}$/#*all*$_{\mathrm{F}}\}$ *of her dinner*). I chose a scale of quantities here because it (arguably) tracks entailment.

should put (38) and (39) in the same kind of competition. Why, then, does (38) not presuppose the negation of the stronger prejacent in (39)?

The answer likely has to do with the fact that (38), by its meaning, *asserts* that the presupposition of (39) is false. So applying the PIP to it is in an obvious sense redundant. Perhaps, then, the PIP should be restated in a way that prevents this redundancy and keeps the principle from applying to pairs like (38) and (39). Such a revision may say that, given a sentence ϕ and an alternative ψ with a stronger true presupposition, *if* ϕ *is consistent with the presupposition of* ψ, then ϕ is infelicitous. Whenever ϕ is *not* consistent with the presupposition of ψ, it follows that ϕ itself entails the falsity of that presupposition. In such cases the PIP adds nothing of its own, and is therefore inapplicable:

(40) **Presupposed Ignorance Principle (PIP) — revised**:
If $\psi \in \mathrm{ALT}(\phi)$ and $Dom(\llbracket\psi\rrbracket^{¢}) \subset Dom(\llbracket\phi\rrbracket^{¢})$, then $*\phi$ if common ground supports $Dom(\llbracket\psi\rrbracket^{¢})$ and $\llbracket\phi\rrbracket^{¢} \cap Dom(\llbracket\psi\rrbracket^{¢}) \neq \varnothing$.

Now we can distinguish (36) from (38). The meaning of (36) says nothing about whether all of the students smoke, so it is consistent with the presupposition of (37). (40) is therefore applicable to (36), and it is predicted to block it whenever all of the students smoke. This is good. (38), on the other hand, *entails* that Kim did not use more than a teaspoon of vinegar, so it is not consistent with the presupposition of its alternative in (39). Therefore (38) is not blocked by the PIP, and we correctly predict its falsity (rather than infelicitousness) in contexts where Kim used more than a teaspoon of vinegar.

I take it that something like (40) is necessary if we want the PIP to interact well with sentences containing *only*.[18] With this in mind, we now return to *only* $(\Box(p \vee q)_{\mathrm{F}})$. If this type of sentence *asserted* the negations of $\Box p, \Box q$, then by its meaning it would not be compatible with the presuppositions of the alternatives *only* $\Box p_{\mathrm{F}}$ and *only* $\Box q_{\mathrm{F}}$. It follows that the revised PIP would not apply to it, for the same reason as in (38). But here this would be bad news, because we want the falsity of $\Box p, \Box q$ to be presupposed, not asserted. So we have not yet succeeded in combining B-L&F's view with the (revised) PIP to derive $\neg\Box p, \neg\Box q$ as presuppositions. The only way to make the combination work is to keep the alternatives $\Box p, \Box q$ out of *only*'s IE set in *only* $(\Box(p \vee q)_{\mathrm{F}})$. This way, their negations will no longer be entailed by the sentence, and the PIP will apply, as desired. But we have already discussed the possibility of making $\Box p, \Box q$ non-IE. We saw that this could be achieved by adding $\Diamond p, \Diamond q$ to *only*'s set of alternatives, and we saw that doing this is problematic: it allows *only*'s alternatives to differ by more than its focus-associate, and it conflicts with Crnič et al.'s (2015) findings about the $\neg\Box p, \neg\Box q$ implicatures of $\Box(p \vee q)$.

We will talk a little more about the PIP in the next section. The conclusion so far is that the principle does not account for the FC-like presuppositions of *only* $\Box(p \vee q)_{\mathrm{F}}$. As things stand, then, we do not know how B-L&F's proposal can fit into a more general account of how *only* interacts with disjunctive foci.

Let us turn to Cases 2 and 3.

[18] We will make a similar move when we spell out the details of how presuppositional implicatures are derived. See Section 4.1.

3.3 Cases 2 and 3: FC with unfocused disjunctions

Like the sentences discussed above, (41a,b) have readings that presuppose FC:

(41) a. Only Kim_{F} is required to eat soup or salad.
  b. Only Kim_{F} is allowed to eat soup or salad.

Sentence (41a) has a reading that presupposes an obligation on Kim's part. It also says that Kim is the only one who has it, and that it is not specifically about soup, nor specifically about salad. We can go further and eliminate the possibility of an ignorance reading. Assume a context where a judge in a cooking show is discussing what the contestants need and do not need to prepare (or what they can and cannot prepare). In one context the judge says (42a); in another s/he says (42b).

(42) a. Only Kim_{F} is required to prepare soup or salad.
  b. Only Kim_{F} is allowed to prepare soup or salad.

If the judge is an authority on the rules, and if in each of these contexts the given sentence is taken to communicate all of the relevant information, then both (42a) and (42b) license an FC inference: $\Box(p \vee q)$ together with $\neg\Box p, \neg\Box q$ in (42a), and $\Diamond p, \Diamond q$ in (42b). (42a) also says that other contestants are not required to prepare either soup or salad ($\neg\Box(p \vee q)$); (42b) says that they have no permission to prepare either ($\neg\Diamond(p \vee q)$).

The question, once again, is how these inferences can be captured on B-L&F's account of *only*. The first thing to notice is that, here, focus-marking appears on the subject *Kim* and nothing else. If this means that the contents of C will be identical to the prejacent in every respect *except* for the focused subject (as in (43a,b)), it follows that no II-alternatives will help derive FC, because none of the alternatives have the disjunction replaced with its disjuncts:

(43) a. In the case of (42a), $C = \{$Chris is required to prepare soup or salad,
  Alex is required to prepare soup or salad, $\cdots\}$
  b. In the case of (42b), $C = \{$Chris is allowed to prepare soup or salad,
  Alex is allowed to prepare soup or salad, $\cdots\}$

Let us discuss this assumption about alternatives briefly. Why should we think that *only*'s alternative-set is constrained as in (43) in such cases? The answer comes from simpler baselines, like (44):

(44) Only Kim_{F} ate soup.

Intuitively (44) asserts that nobody other than Kim ate soup. But if an alternative could be generated where the unfocused *soup* is replaced with some salient alternative like *salad*, we expect that *only* also assert that Kim ate nothing but soup, by negating e.g. the generated alternative [*Kim ate salad*]. This is not the case however, as the contrast below shows:

(45) a. Chris and many others ate soup, #but only Kim_{F} ate soup.
  (second clause entails that [*X ate soup*] is false for all $X \neq Kim$)

b. Kim ate salad and many other things, ✓but only s/he$_F$ ate soup.
(second clause does not entail that [*Kim ate X*] is false for all $X \neq soup$)

Perhaps in (45) the alternatives do not come up because they require replacement rather than simplification of unfocused material. But (46) shows that simplification (of unfocused material) is also not possible in generating alternatives to *only*:

(46) Only Kim$_F$ can't eat soup and salad.

Here replacing the unfocused conjunction with its conjuncts is predicted to lead (incorrectly) to an FC assertion. Suppose alternatives can be generated by leaving *Kim* unchanged but replacing [*soup and salad*] with [*soup*] and with [*salad*]. Then we get the alternatives [*Kim can't eat soup*] and [*Kim can't eat salad*], whose negations are consistent with the prejacent, and which (when negated) should lead to the inference that Kim is allowed to eat soup and allowed to eat salad. The sentence does not assert this however.[19]

Let us now return to (43) and the sentences in (42). Unlike Case 1 from earlier, (42a) can be explained by the (revised) PIP: the principle blocks the sentence if it has an alternative with a stronger true presupposition. Assuming both of (47a,b) to be alternatives, and given that they carry stronger presuppositions than (42a), it follows by the PIP that the felicitousness of (42a) requires that the presuppositions of (47a,b) not be met in context: it must not be common ground that Kim needs to prepare soup, and it must not be common ground that s/he needs to prepare salad.

(47) a. Only Kim$_F$ is required to prepare soup.
b. Only Kim$_F$ is required to prepare salad.

But the PIP cannot derive the FC presupposition in the case of (42b). In fact, if we maintain the PIP along with our assumptions about alternatives, we predict (42b) to be infelicitous in FC scenarios. The reason is parallel to the case of (42a): by assumption, (42b) has the following formal alternatives:

(48) a. Only Kim$_F$ is allowed to prepare soup.
b. Only Kim$_F$ is allowed to prepare salad.

Each of the alternatives in (48) has a stronger presupposition than (42b). It follows that (42b) should be infelicitous in contexts that support the presuppositions of (48a,b). So unlike (42a), (42b) shouldn't have a reading that presupposes FC; it should suggest ignorance about what Kim is allowed to prepare. But intuitively (42a,b) do not differ in this way —

[19] Attentive readers will note that if we also admit alternatives where the possibility modal is replaced with a necessity modal, the incorrect assertion goes away, since none of the alternatives are innocently excludable in that case. But remember that we are testing the possibility of generating alternatives to *only* where unfocused material can be simplified but *not replaced*.

they are alike in the availability of their respective FC presuppositions.[20,21]

Let me summarize the claims so far. First, sentences of the form *only*$(\Box(p \vee q)_{\text{F}})$ presuppose $\neg\Box p, \neg\Box q$. The presupposition cannot be derived on B-L&F's account of *only*, because the disjunct-alternatives to $\Box(p \vee q)_{\text{F}}$ — $\Box p$ and $\Box q$ — are innocently excludable. Their negations should therefore be part of *only*'s assertion rather than its presupposition. Second, appeal to Spector and Sudo's PIP to derive these presuppositions, though possible at first glance, does not work unless it is accompanied by unwanted assumptions. Third, cases of the form [*only* $\text{X}_{\text{F}}(\Box/\Diamond(p \vee q))$], where *only*'s prejacent contains an unfocused modalized disjunction, also have FC presuppositions. These, however, cannot be derived on B-L&F's account, and the PIP produces divergent results for them.

This makes the bulk of my empirical points. I take the data to suggest that something other than *only*'s meaning is behind the FC presuppositions observed in Cases 1-3. In the next section I will say what I think the responsible mechanism is, and what it implies for B-L&F's account.

4 The proposal

Our data show a familiar pattern: FC is licensed in *only*'s presupposition but not in its assertion. The perspective that makes this familiar comes from Gajewski and Sharvit (2012), who noted that scalar implicatures appear systematically, and seemingly independently, in the upward-entailing presupposition of what may otherwise be a downward-entailing context.[22] The presuppositional implicature of *some* in (49) shows this:[23]

(49) Kim is unaware that some of our students smoke.
Presupposition: Some (but not all) of our students smoke.
Assertion: Kim's belief state allows that none of our students smoke.

As (49) shows, the scalar item *some* is strengthened in the factive presupposition of the predicate *unaware*, but not in its negative assertion about Kim's belief state. *Only*, like *unaware*, has a positive presupposition and a negative assertion, and when *some* appears (unfocused) under *only*, its 'not-all' implicature is calculated in the particle's (positive) presupposition but not in its (negative) assertion:

[20] Marty and Romoli (2020) directed a similar argument against the PIP in general, using sentences like (i) where FC is intuitively presupposed, but not predicted to be, by the PIP:

(i) Kim is unaware that Mel is allowed to eat soup or salad.
Presupposition: Mel is allowed to eat soup and allowed to eat salad. (FC)
Assertion: Kim allows the possibility that Mel has no permission to eat either.

We will see more of Marty and Romoli in Section 4.2.

[21] See footnote 16 for an explanation why this problem of the PIP did not come up when we looked at focused disjunctions, i.e. in the cases of *only*$\Box(p \vee q)_{\text{F}}$ and *only*$\Diamond(p \vee q)_{\text{F}}$.

[22] Related points were made earlier by Russell (2006) and Simons (2006), responding in turn to Chierchia (2004).

[23] Example (49) is identical to (36) from Section 3.2. At that time I did not refer to the 'not-all' inference of *some* as a presuppositional implicature, but used it to introduce Spector and Sudo's PIP. From this point on I will continue to talk about these inferences as (presuppositional) implicatures, in light of the challenges to the PIP reviewed above. See Marty and Romoli 2020 for detailed discussion.

(50) Only Kim$_{\mathrm{F}}$ did some of the readings.
Presupposition: Kim did some (but not all) of the readings.
Assertion: Others did not do any of the readings.

It follows then that, if FC is a scalar implicature, it should appear in *only*'s presupposition in (1) — this point was made in Alxatib 2014, but it was specific to (1). More generally, however, we expect to see FC in any context where implicatures are licensed, including contexts where implicatures appear to be calculated in the presupposition separately from the assertion. How this can be implemented is the concern of this section.

I begin by making my first assumption explicit: when *only* takes a disjunctive associate $[p \vee q]$, it does not operate on alternatives where $[p \vee q]$ is replaced with p or with q. This assumption does not fit very well with current literature, nor indeed with the assumption that, to Exh, p, q are in fact alternatives to $[p \vee q]$. For the time being my main motivation for taking this view is empirical: it is based on the finding from Section 3.1 that sentences of the form $only(\Box(p \vee q)_{\mathrm{F}})$ do not assert the negations of the alternatives $\Box p$ and $\Box q$, while they do assert the negation of $\Box r$. With this said, I must emphasize that this assumption does not by itself constitute a revision to how we think alternatives are generated; it likely has more to do with what *only* requires of its alternatives. One possibility is that *only* requires that, whenever one of its formal alternatives entails the prejacent, then the alternative must stand in an entailment relation with every other alternative. This can never be true of disjuncts given a disjunctive prejacent, because the disjuncts do not entail one another, but they each entail the prejacent.[24] On the other hand, alternatives that are independent of the disjunction satisfy the requirement, as long as they do not entail each other.

I will leave further development of this assumption to future work. Let me write it in a specific form and a possibly more general form for now:

(51) **Specific generalization about disjunctive associates to *only***:
In $[only_C\ [\cdots [A\ or\ B\,]_{\mathrm{F}} \cdots]]$, C does not contain an alternative where [*A or B*] is replaced with *A*, nor an alternative where [*A or B*] is replaced with *B*.

(52) **Constraint on C in $only_C$**:
p is in the domain of $only_C$ only if
$\forall q(q \in C \,\&\, q \subset p \rightarrow (\forall r(r \in C \,\&\, r \neq q \rightarrow (q \subset r \vee r \subset q))))$[25]

Both (51) and (52) block membership of $\Box p, \Box q$ and $\Diamond p, \Diamond q$ in C, where $only_C$ takes the prejacents $\Box(p \vee q)_{\mathrm{F}}$, $\Diamond(p \vee q)_{\mathrm{F}}$ respectively.[26] If this is right, then *only* has no access to any II-alternatives in the latter case, so B-L&F's account of FC under *only* would not work. The results we get are the following:

[24] I am not considering so-called "Hurford" disjunctions like [*in Paris or in France*] (Hurford 1974), which are independently problematic.

[25] Other, similar formulations are conceivable. It could be that $only_C\, p$ is defined only if $\{p\} \cup C$ is either totally ordered by entailment or unordered by entailment. Note that the condition (on either version) can in principle be generalized to all scales, logical and otherwise. But I will not get into this here.

[26] Strictly speaking, neither condition blocks the conjunctive alternative: (51) says nothing about it; (52) blocks it in the presence of independent alternatives $\Box r / \Diamond r$ in C. If my claim in footnote 3 is correct, this result would have to be looked at more carefully.

(53) Because $\Box p, \Box q \notin C$,
$only_C \Box(p \vee q)_F$ presupposes $\Box(p \vee q)$ and asserts $\neg\Box r$ & $\neg\Box(p \wedge q)$.

(54) Because $\Diamond p, \Diamond q \notin C$,
$only \Diamond(p \vee q)_F$ presupposes $\Diamond(p \vee q)$ and asserts $\neg\Diamond r$ & $\neg\Diamond(p \wedge q)$.

These results are correct in blocking $\neg\Box p, \neg\Box q$ from *only*'s assertion in (53), but they fall short of deriving it as a presupposition. They also fall short of deriving the FC presupposition in (54). I will now show how these inferences can be derived uniformly as presuppositional implicatures.

4.1 The proposal (the simple version)

The first way of capturing the facts above is inspired by Gajewski and Sharvit 2012, though here I will follow Marty (2017) and execute the idea slightly differently. Let us assume that Exh determines not only the truth/falsity of its prejacent and its alternatives, but determines also a *domain* (presupposition) in terms of the domain of the prejacent and those of its alternatives. I remind the reader that $Dom(\phi)$ is to be read as "the presupposition of ϕ".

Let us now add a little more formalism. In the rest of the paper I will occasionally talk about the presuppositions of a *set* of propositions. Given such a set A, I will write A^{Dom} when I intend the set that contains the presuppositions of A's elements. That is:

(55) For any set of propositions A, that is, a set A whose elements are (partial) functions from worlds to truth values,
$A^{Dom} = \{\xi : \exists\psi(\psi \in A \text{ \& } \xi = Dom(\psi))\}$

Now we write that the implicature-enriched presupposition of ϕ, given C, is the result of applying Exh to the presupposition of ϕ, relative to C^{Dom}, i.e. the presuppositions of its alternatives:

(56) **Definedness condition on Exh – DOM-EXH (Take 1)**:
$Dom(\text{Exh}(C)(\phi)) \subseteq \text{Exh}(C^{Dom})(Dom(\phi))$

This first version of DOM-EXH gives us some basic results, though soon we will see reasons — familiar ones — to write a second version. One example where it seems to work on first pass is (57), where the scalar term *some* is embedded under the factive verb *know*.

(57) Kim knows that some of our students smoke.

We may assume that (57) has (58) as a formal alternative:

(58) Kim knows that all of our students smoke.

And given DOM-EXH, it follows that exhaustifying (57) produces the presupposition that some, but not all, of our students smoke. This comes from applying Exh to the domain of (57), i.e. the proposition that some students smoke, given the (singleton) set that contains the presupposition of (58). I summarize this in (59), with $K\exists/K\forall$ as shorthands for the denotations of (57) and (58).

(59) $Dom(\text{Exh}(\{K\forall\})(K\exists))$
$\subseteq \text{Exh}(\{K\forall\}^{Dom})(Dom(K\exists))$, i.e.
$\subseteq \text{Exh}(\{\forall\})(\exists)$, i.e.
$\subseteq \exists \,\&\, \neg\forall$

The same result is predicted when *know* is replaced with the Strawson-DE predicate *unaware*, given its alternative in (61) — (60) is repeated from (49) and (36) above. I take the meaning of *unaware that S* to be the same as *does not know that S*, hence the abbreviation $\neg K$ in (60)-(62).

(60) Kim is unaware that some of our students smoke. $(\neg K\exists)$

(61) Kim is unaware that all of our students smoke. $(\neg K\forall)$

(62) $Dom(\text{Exh}(\{\neg K\forall\})(\neg K\exists))$
$\subseteq \text{Exh}(\{\neg K\forall\}^{Dom})(Dom(\neg K\exists))$, i.e.
$\subseteq \text{Exh}(\{\forall\})(\exists)$, i.e.
$\subseteq \exists \,\&\, \neg\forall$

So from the first formulation of the definedness condition on Exh, we derive the implicature-strengthening of the *some* presupposition in (57) and (60). But while at first glance the two sentences appear to be alike in this respect, Spector and Sudo pointed out that the presuppositional implicature is stronger in the second case than it is in the first. We will return to this issue shortly.[27] At the moment I want to make sure that DOM-EXH works with *only*. We are about to see a similar discussion to the one we had in the context of the PIP (Section 3.2). Take (63), repeated from (38).

(63) Kim only used [a teaspoon]$_{\text{F}}$ of vinegar.

Assume that (63) has (64) (=(39)) as a stronger formal alternative:[28]

(64) Kim only used [a tablespoon]$_{\text{F}}$ of vinegar.

Then it follows from DOM-EXH that, upon exhaustification, (63) should *presuppose* that Kim used no more than a teaspoon of vinegar. In fact the sentence says this in its assertion. So we want to narrow the reach of DOM-EXH just like we did when we revised the PIP earlier. I propose that in cases like these, the problematic alternatives are ignored in DOM-EXH:

(65) **Definedness condition on Exh (Take 2) – DOM-EXH2**:
$Dom(\text{Exh}(C)(\phi)) \subseteq \text{Exh}(C^{Dom}_{*,\phi})(Dom(\phi))$,
where $C_{*,\phi} = \{\psi : \psi \in C \,\&\, \phi \cap Dom(\psi) \neq \varnothing\}$,
and $C^{Dom}_{*,\phi} = \{\xi : \exists\psi(\psi \in C_{*,\phi} \,\&\, \xi = Dom(\psi))\}$

[27] Russell (2006) also pointed out the weakness of the putative implicature in the case of *know*, contrary to what would be predicted from DOM-EXH. He also discussed an example where *know* is negated, but did not compare the strength of the presuppositional implicature in the two cases. See his (12), shown below as (i):

(i) George doesn't know that failure in Iraq is possible.
Implies: Failure in Iraq is not necessary.

[28] See footnote 17 for comments about the use of quantity expressions in these examples.

In words, the presupposition of ϕ, upon exhaustification with respect to a set of alternatives C, is whatever results from exhaustifying the presupposition of ϕ against the presuppositions of a *subset* of C. That subset comes from keeping from C only those propositions whose domain is compatible with ϕ; alternatives that fail this requirement have presuppositions that can't be true given ϕ — ϕ by itself entails the falsity of their presuppositions. (64) is an example: its presupposition (that Kim used a tablespoon of vinegar) is not compatible with the assertion of (63), which says that s/he did not use more than a teaspoon. By DOM-EXH2, (64) and other alternatives with a stronger prejacent to *only* will be removed from consideration in determining the presupposition of Exh(63). So, assuming that ϕ=*only* [a teaspoon]$_{\mathrm{F}}$, and C={*only* [a tablespoon]$_{\mathrm{F}}$, *only* [a fluid ounce]$_{\mathrm{F}}, \cdots$}, then:

(66) a. $C_{*,\phi} = \varnothing$

b. $Dom(\mathrm{Exh}(C)(\phi))$
$\subseteq \mathrm{Exh}(C_{*,\phi}^{Dom})(Dom(\phi))$, i.e.
$\subseteq \mathrm{Exh}(\{\}^{Dom})(\mathrm{tsp})$, i.e.
$\subseteq \mathrm{tsp}$

The revision from DOM-EXH to DOM-EXH2 is just like the one proposed for the PIP earlier, and here too I think there is an argument to be made for it. If exhaustification, by its design, adds information beyond what its argument says semantically, then it would be uneconomical if there were instances where Exh essentially duplicates what its argument already entails. (63) is an example of that.

We are now ready to derive our FC presuppositional implicatures.

4.1.1 Case 1 and its parallel with possibility modals

Case 1 is (67), repeated from (22):

(67) Kim only needs to eat [soup or salad]$_{\mathrm{F}}$. $only_C \Box(p \vee q)_{\mathrm{F}}$
Target presupposition: Kim does not need to eat soup, $\neg\Box p$
Kim does not need to eat salad. $\neg\Box q$

Recall first the result derived earlier, that $only_C \Box(p \vee q)_{\mathrm{F}}$ itself presupposes $\Box(p \vee q)$ and asserts $\neg\Box r \,\&\, \neg\Box(p \wedge q)$. This came from keeping the alternatives $\Box p, \Box q$ out of C in calculating the literal meaning of $only_C \Box(p \vee q)_{\mathrm{F}}$.

Now let us assume that (67) has the formal alternatives in (68):

(68) {Kim only needs to eat soup$_{\mathrm{F}}$, $only(\Box p_{\mathrm{F}})$
Kim only needs to eat salad$_{\mathrm{F}}$, $only(\Box q_{\mathrm{F}})$
Kim only needs to eat rice$_{\mathrm{F}}$, $only(\Box r_{\mathrm{F}})$
Kim only needs to eat [soup and salad]$_{\mathrm{F}}, \cdots$} $only(\Box(p \wedge q)_{\mathrm{F}})$

Of these, the third and fourth alternatives will be removed from consideration by DOM-EXH2, because their presuppositions ($\Box r$ and $\Box(p \wedge q)$) conflict with the literal meaning of (67). This is not the case of the first two alternatives, however, so their presuppositions are predicted to participate in enriching (67):

(69) Let ϕ=(67), C=(68). Then:

a. $C_{*,\phi} = \{only(\Box p_{\mathrm{F}}), only(\Box q_{\mathrm{F}})\}$

b. $C^{Dom}_{*,\phi} = \{\Box p, \Box q\}$

c. $Dom(\mathrm{Exh}(C)(\phi)) \subseteq \mathrm{Exh}(C^{Dom}_{*,\phi})(Dom(\phi))$, i.e.
$\subseteq \mathrm{Exh}(\{\Box p, \Box q\})(Dom(only\Box(p\vee q)_{\mathrm{F}}))$, i.e.
$\subseteq \mathrm{Exh}(\{\Box p, \Box q\})(\Box(p\vee q))$, i.e.
$\subseteq \Box(p\vee q)\ \&\ \neg\Box p\ \&\ \neg\Box q$

And if we follow B-L&F and add an II component to Exh, the original case of (1), repeated as (70), follows straightforwardly.

(70) Kim is only allowed to eat [soup or salad]$_{\mathrm{F}}$. $\quad only_C\Diamond(p\vee q)_{\mathrm{F}}$
Target presupposition: Kim is allowed to eat soup, $\quad \Diamond p$
Kim is allowed to eat salad. $\quad \Diamond q$

By our assumptions, (70) presupposes $\Diamond(p\vee q)$ and asserts $\neg\Diamond r$ and $\neg\Diamond(p\wedge q)$. Its alternatives parallel those of Case 1, with $\Diamond$ taking the place of $\Box$:

(71) {Kim is only allowed to eat soup$_{\mathrm{F}}$, $\quad only(\Diamond p_{\mathrm{F}})$
Kim is only allowed to eat salad$_{\mathrm{F}}$, $\quad only(\Diamond q_{\mathrm{F}})$
Kim is only allowed to eat rice$_{\mathrm{F}}$, $\quad only(\Diamond r_{\mathrm{F}})$
Kim is only allowed to eat [soup and salad]$_{\mathrm{F}}$, $\cdots$} $\quad only(\Diamond(p\wedge q)_{\mathrm{F}})$

And again, by DOM-EXH2, the bottom two alternatives are overlooked in determining the presupposition of Exh(70), with the result in (72c). This time the FC inference comes from the II mechanism of Exh.

(72) Let ϕ=(70), C=(71). Then:

a. $C_{*,\phi} = \{only(\Diamond p_{\mathrm{F}}), only(\Diamond q_{\mathrm{F}})\}$

b. $C^{Dom}_{*,\phi} = \{\Diamond p, \Diamond q\}$

c. $Dom(\mathrm{Exh}(C)(\phi)) \subseteq \mathrm{Exh}(C^{Dom}_{*,\phi})(Dom(\phi))$, i.e.
$\subseteq \mathrm{Exh}(\{\Diamond p, \Diamond q\})(Dom(only\Diamond(p\vee q)_{\mathrm{F}}))$, i.e.
$\subseteq \mathrm{Exh}(\{\Diamond p, \Diamond q\})(\Diamond(p\vee q))$, i.e.
$\subseteq \Diamond(p\vee q)\ \&\ \Diamond p\ \&\ \Diamond q$

Interim summary and consequences. The target inferences of the sentences *only* $\Box(p\vee q)_{\mathrm{F}}$ and *only* $\Diamond(p\vee q)_{\mathrm{F}}$ were derived uniformly as presuppositional implicatures. A crucial part of the proposal is the assumption that *only* has no access to the disjunct-alternatives when its associate in the prejacent is a disjunction. If this is right, it would mean (contra B-L&F) that there is no evidence that *only* makes reference to II-alternatives in its presupposition, because if disjuncts aren't visible to *only* in the first place, reference to II-alternatives would be vacuous. Conversely, if disjuncts were visible to *only*, we would predict that $\neg\Box p$ and $\neg\Box q$ be part of *only*'s assertion, something that was argued in Section 3.1 to be incorrect, and to not be amenable to repair either by Spector and Sudo's PIP or by admitting into C (in $only_C$) alternatives where unfocused material is modified. If the alternatives $\Box p, \Box q$ are not visible to *only* in *only* $\Box(p \vee q)_{\mathrm{F}}$, their analogs $\Diamond p, \Diamond q$ should likewise not be visible to it in the case of *only* $\Diamond(p \vee q)_{\mathrm{F}}$.[29]

4.1.2 Cases 2 and 3: unfocused disjunction

Cases 2 and 3 are repeated below (from (41)):

(73) Only Kim$_{\mathrm{F}}$ is required to eat soup or salad. $\quad only_C Kim_{\mathrm{F}} \Box(p \vee q)$
Target presupposition: Kim is not required to eat soup, $\quad \neg Kim \Box p$
Kim is not required to eat salad. $\quad \neg Kim \Box q$

(74) Only Kim$_{\mathrm{F}}$ is allowed to eat soup or salad. $\quad only_C Kim_{\mathrm{F}} \Diamond(p \vee q)$
Target presupposition: Kim is allowed to eat soup, $\quad Kim \Diamond p$
Kim is allowed to eat salad. $\quad Kim \Diamond q$

These two cases have the same account on the current proposal. Let us assume that (73) has alternatives where the associate is replaced with alternatives to *Kim*, and also where the unfocused disjunction is replaced with its own alternatives. This gives us:

(75) {Only Kim$_{\mathrm{F}}$ is required to eat soup, $\quad [only\ Kim_{\mathrm{F}} \Box(p)]$
Only Kim$_{\mathrm{F}}$ is required to eat salad, $\quad [only\ Kim_{\mathrm{F}} \Box(q)]$
Only Kim$_{\mathrm{F}}$ is required to eat soup and salad, $\quad [only\ Kim_{\mathrm{F}} \Box(p \wedge q)]$
Only Mel$_{\mathrm{F}}$ is required to eat soup or salad, $\quad [only\ Mel_{\mathrm{F}} \Box(p \vee q)]$
Only Mel$_{\mathrm{F}}$ is required to eat soup, $\quad [only\ Mel_{\mathrm{F}} \Box(p)]$
Only Mel$_{\mathrm{F}}$ is required to eat salad, $\cdots$} $\quad [only\ Mel_{\mathrm{F}} \Box(q)]$

The last three alternatives in this set, where *Mel* replaces *Kim*, have presuppositions that conflict with the assertion of (73). For this reason they are overlooked by DOM-EXH2. Thus the enrichment of (73) presupposes the result of exhaustifying its presupposition, $Kim \Box(p \vee q)$, against those of the top three alternatives in (75). This gives us the target presuppositional implicature, as summarized below.

(76) Let $\phi = (73)$, $C = (75)$. Then:

a. $C_{*,\phi} = \{[only\ Kim_{\mathrm{F}} \Box(p)], [only\ Kim_{\mathrm{F}} \Box(q)], [only\ Kim_{\mathrm{F}} \Box(p \wedge q)]\}$

b. $C^{Dom}_{*,\phi} = \{Kim \Box p, Kim \Box q, Kim \Box(p \wedge q)\}$

c. $Dom(\mathrm{Exh}(C)(\phi))$
$\subseteq \mathrm{Exh}(C^{Dom}_{*,\phi})(Dom(\phi))$, i.e.
$\subseteq \mathrm{Exh}(\{Kim \Box p, Kim \Box q, Kim \Box(p \wedge q)\})(Dom(only\ Kim_{\mathrm{F}} \Box(p \vee q)))$, i.e.
$\subseteq \mathrm{Exh}(\{Kim \Box p, Kim \Box q, Kim \Box(p \wedge q)\})(Kim \Box(p \vee q))$, i.e.
$\subseteq Kim \Box(p \vee q)\ \&\ \neg Kim \Box p\ \&\ \neg Kim \Box q$

Case 3, instantiated in (74), behaves in the same way, with the difference that its FC presuppositional implicature results from Exh's II component. The alternatives and derivation are shown in (77) and (78). The alternatives ignored by DOM-EXH2 are struck out.

[29] This conclusion has a further consequence, for it also shows that *only* need not make reference to IE-alternatives! I return to this in the conclusion.

(77) $\{$Only Kim$_F$ is allowed to eat soup, $[only\ Kim_F\Diamond(p)]$
Only Kim$_F$ is allowed to eat salad, $[only\ Kim_F\Diamond(q)]$
Only Kim$_F$ is allowed to eat soup and salad, $[only\ Kim_F\Diamond(p\wedge q)]$
~~Only Mel$_F$ is allowed to eat soup or salad~~, $[only\ Mel_F\Diamond(p\vee q)]$
~~Only Mel$_F$ is allowed to eat soup~~, $[only\ Mel_F\Diamond(p)]$
~~Only Mel$_F$ is allowed to eat salad~~, $\cdots\}$ $[only\ Mel_F\Diamond(q)]$

(78) Let ϕ=(74), C=(77). Then:

a. $C_{*,\phi} = \{[only\ Kim_F\Diamond(p)], [only\ Kim_F\Diamond(q)], [only\ Kim_F\Diamond(p\wedge q)]\}$

b. $C^{Dom}_{*,\phi} = \{Kim\Diamond p, Kim\Diamond q, Kim\Diamond(p\wedge q)\}$

c. $Dom(\text{Exh}(C)(\phi))$
$\subseteq \text{Exh}(C^{Dom}_{*,\phi})(Dom(\phi))$, i.e.
$\subseteq \text{Exh}(\{Kim\Diamond p, Kim\Diamond q, Kim\Diamond(p\wedge q)\})(Dom(only\ Kim_F\Diamond(p\vee q)))$, i.e.
$\subseteq \text{Exh}(\{Kim\Diamond p, Kim\Diamond q, Kim\Diamond(p\wedge q)\})(Kim\Diamond(p\vee q))$, i.e.
$\subseteq Kim\Diamond(p\vee q)\ \&\ Kim\Diamond p\ \&\ Kim\Diamond q$

4.2 The less simple version

Any account of presuppositional implicatures must explain the asymmetry mentioned above, discussed explicitly in Spector and Sudo 2017 but related to earlier observations in Russell 2006 and Simons 2006. The paradigm is repeated below.

(79) a. Kim knows that some of our students smoke. $K\exists$
$\not\Rightarrow$ Not all of our students smoke. $\not\Rightarrow \neg\forall$

b. Kim is unaware that some of our students smoke. $\neg K\exists$
$\Rightarrow$ Not all of our students smoke. $\Rightarrow \neg\forall$

Our formulation of DOM-EXH2 does not distinguish the status of the $\neg\forall$ inference in these two cases, so the current proposal about *only* and its FC inference must be revisited once we have a view that accounts for (79a-b). For this, I turn to Marty and Romoli 2020.

Marty and Romoli (M&R) reduce the asymmetry in (79) to a condition on alternative pruning. In brief, the story is as follows. The $\neg\forall$ inference is cancellable in (79a) because it does not come from global exhaustification. Global exhaustification, in fact, generates the opposite inference $\forall$. When the responsible alternative is pruned, that inference is suspended, and the possibility of the $\neg\forall$ inference arises. In (79b), global exhaustification does generate the $\neg\forall$ inference, and in this case the responsible alternative may not be pruned.

The conditions that produce this difference are built on a view that divides alternatives into different types: some alternatives are "presuppositional"; others are "assertoric". The (innocent) excludability of a presuppositional alternative depends on the (innocent) excludability of its *presupposition* given the prejacent and the presuppositions of the other presuppositional alternatives. When such an alternative is IE, the negation of its presupposition is inherited by the prejacent upon exhaustification. The excludability of an assertoric alternative depends on the excludability of its overall meaning, relative to the prejacent, as enriched by IE presuppositional alternatives and the remaining assertoric alternatives. When such an alternative is IE, its presupposition is inherited upon exhaustification, and its assertion is added to the enriched meaning of the prejacent. Innocent Includability is divided in a similar way, but I will return to it after we assemble the pieces that we have so far. We will see how M&R's division of alternatives helps distinguish the presuppositions of (79a) and (79b).

A formal alternative ψ is presuppositional, given a proposition ϕ and a set of alternative propositions C, iff ψ is Strawson-entailed by ϕ but is not logically entailed by it:[30]

(80) **Presuppositional alternatives**:
Given a proposition ϕ and set of alternative propositions C,
$\text{Pr}(C)(\phi) = \{\psi : \psi \in C \;\&\; \phi \nvDash \psi \;\&\; \phi \vDash_S \psi\}$

With this classification we already introduce a difference into (79). The proposition $K\forall$ is not a logical consequence of $K\exists$, and it is not a Strawson-consequence of it either. So $K\forall$ cannot be a presuppositional alternative to (79a). By contrast, $\neg K\forall$ is a Strawson-consequence of $\neg K\exists$, even though it does not follow from it logically. Therefore the $\forall$-alternative $\neg K\forall$ qualifies as a presuppositional alternative in (79b). This result is written in (79′).

(79′) a. For $\phi = K\exists$ and $C = \{K\forall\}$, $\text{Pr}(C)(\phi) = \{\}$
b. For $\phi = \neg K\exists$ and $C = \{\neg K\forall\}$, $\text{Pr}(C)(\phi) = \{\neg K\forall\}$

Now let us select whatever is IE from the presuppositions of the Pr sets above. The selection is vacuous in (79′a), but in (79′b) we find that the sole member of $\text{Pr}(C)(\phi)$ has an IE presupposition relative to ϕ and C. Let us define $\text{IE}_{\textbf{Pr}}(C)(\phi)$ as that subset of $\text{Pr}(C)(\phi)$ whose elements have IE presuppositions given one another and given ϕ:

(81) $\text{IE}_{\textbf{Pr}}(C)(\phi) = \{\psi : \psi \in \text{Pr}(C)(\phi) \;\&\; Dom(\psi) \in \text{IE}(\text{Pr}^{Dom}(C)(\phi))(\phi)\}$,
where $\text{Pr}^{Dom}(C)(\phi) = \{\pi : \exists\psi(\psi \in \text{Pr}(C)(\phi) \;\&\; \pi = Dom(\psi))\}$

In our example this turns up a simple result:

(79″) a. For $\phi = K\exists$ and $C = \{K\forall\}$, $\text{IE}_{\textbf{Pr}}(C)(\phi) = \{\}$
b. For $\phi = \neg K\exists$ and $C = \{\neg K\forall\}$, $\text{IE}_{\textbf{Pr}}(C)(\phi) = \{\neg K\forall\}$

Now we come to our first version of DOM-EXH in M&R's proposal. The domain of ϕ, on exhaustification w.r.t. C, is a subset of the domain of ϕ and is disjoint from the domains of ϕ's $\text{IE}_{\textbf{Pr}}$ alternatives:

(82) **Definedness condition on Exh (Take 3) – DOM-EXH3**:
$Dom(\text{Exh}(C)(\phi)) \subseteq Dom(\phi) - \bigcup \text{IE}_{\textbf{Pr}}^{Dom}(C)(\phi)$,
where $\text{IE}_{\textbf{Pr}}^{Dom}(C)(\phi) = \{\pi : \exists\psi(\psi \in \text{IE}_{\textbf{Pr}}(C)(\phi) \;\&\; \pi = Dom(\psi))\}$

This gives us the $\neg\forall$ presupposition for (79b), but no presuppositional implicatures yet for (79a):

(79‴) a. For $\phi = K\exists$ and $C = \{K\forall\}$, $Dom(\text{Exh}(C)(\phi)) \subseteq \exists$

[30] ϕ Strawson-entails ψ iff $\phi \cap Dom(\psi) \subseteq \psi$.

b. For $\phi = \neg K\exists$ and $C = \{\neg K\forall\}$, $Dom(\text{Exh}(C)(\phi)) \subseteq (\exists - \forall)$

Next we turn to the assertoric alternatives. Defining these is straightforward, but we must first revisit our definition of Innocent Excludability and be explicit about how presupposition-carrying alternatives should be handled. We will say that ψ is IE, given a proposition ϕ and set of alternatives C, iff its *strong* negation, $\neg\psi$, does not affect the consistency of any selection of *strong* exclusions from C. By "strong negation of ψ" we mean the proposition that ψ is defined and false. The exclusion of an IE-alternative therefore projects the presupposition of that alternative.

(83) $\text{IE}(C)(\phi) = \{\psi : \psi \in C \;\&\; \forall B(B \subseteq C \;\&\; (B^{\neg} \wedge \phi \nvDash \bot) \rightarrow (B^{\neg} \wedge \phi \wedge \neg\psi \nvDash \bot))\}$[31]

Now we identify the assertoric IE-alternatives, $\text{IE}_{\mathbf{A}}$, as those that are IE with respect to C and the enrichment of ϕ that results from applying DOM-EXH3. That is, we make the contribution of assertoric alternatives sensitive to ϕ after its presupposition is strengthened by DOM-EXH3. Let us write $\phi_{+\text{IE}_{\mathbf{Pr}}(C)}$ as shorthand for ϕ together with the enriched presupposition from $\text{IE}_{\mathbf{Pr}}$:

(84) $\phi_{+\text{IE}_{\mathbf{Pr}}(C)} = [\lambda w : w \in (Dom(\phi) - \bigcup \text{IE}^{Dom}_{\mathbf{Pr}}(C)(\phi)) \,.\, \phi(w)]$

We can then define $\text{IE}_{\mathbf{A}}$ as follows:

(85) $\text{IE}_{\mathbf{A}}(C)(\phi) = \text{IE}(C)(\phi_{+\text{IE}_{\mathbf{Pr}}(C)})$

By this definition, the $\forall$-alternative to (79a) will be IE with respect to it: the (strong) negation of $K\forall$ is consistent with $K\exists$ – together the two propositions say that all of our students smoke ($\forall$), and that Kim believes that some do without believing that all do. $K\forall$ is therefore $\text{IE}_{\mathbf{A}}$ with respect to $K\exists$. (79b) has no $\text{IE}_{\mathbf{A}}$ alternatives, however.

(79$''''$) a. For $\phi = K\exists$ and $C = \{K\forall\}$, $\text{IE}_{\mathbf{Pr}}(C)(\phi) = \{\}$, $\text{IE}_{\mathbf{A}}(C)(\phi) = \{K\forall\}$
b. For $\phi = \neg K\exists$ and $C = \{\neg K\forall\}$, $\text{IE}_{\mathbf{Pr}}(C)(\phi) = \{\neg K\forall\}$, $\text{IE}_{\mathbf{A}}(C)(\phi) = \{\}$

We now add the contribution of $\text{IE}_{\mathbf{A}}(C)(\phi)$ alternatives to DOM-EXH: If ψ is $\text{IE}_{\mathbf{A}}$ with respect to ϕ, C, then $\text{Exh}(C)(\phi)$'s domain must be a subset of that of ψ. In short, a proposition inherits the presuppositions of its $\text{IE}_{\mathbf{A}}$ alternatives upon exhaustification:

(86) **Definedness condition on Exh (Take 4) – DOM-EXH4**:
$$Dom(\text{Exh}(C)(\phi)) \subseteq Dom(\phi) \cap \bigcap \text{IE}^{Dom}_{\mathbf{A}}(C)(\phi) - \bigcup \text{IE}^{Dom}_{\mathbf{Pr}}(C)(\phi)$$

By DOM-EXH4, we derive a more dramatic difference between (79a) and (79b): the former is predicted to presuppose $\forall$, by the strong negation of its $\text{IE}_{\mathbf{A}}$ alternative; the latter is predicted to presuppose $\neg\forall$, by the negation of the *presupposition* of its $\text{IE}_{\mathbf{Pr}}$ alternative. As I mentioned earlier, however, M&R account for the finding in (79) by appealing to alternative pruning. In this, they draw on Magri 2009 and propose the following condition: an alternative may not be pruned from $\text{IE}_{\mathbf{A}}$ if its assertion is contextually equivalent to Exh's prejacent, and may not be pruned from $\text{IE}_{\mathbf{Pr}}$ if its presupposition is contextually equivalent to that of Exh's prejacent.

[31] By the same reasoning as in footnote 2, it follows from the updated definition of IE that alternatives that are Strawson-entailed by ϕ will not be IE with respect to it; the empty set of exclusions is consistent with ϕ, but if the strong negation of a Strawson-consequence of ϕ is added, a contradiction follows.

(87) **Condition on alternative pruning**:[32]
Given proposition ϕ, set of alternatives C, alternative $\psi \in C$, and common ground G,
  a. ψ may not be pruned from $\mathrm{IE}_{\mathbf{A}}(C)(\phi)$ if $G \cap \phi = G \cap \psi$,
  b. ψ may not be pruned from $\mathrm{IE}_{\mathbf{Pr}}(C)(\phi)$ if $G \cap Dom(\phi) = G \cap Dom(\psi)$.

This condition completes M&R's account of the asymmetry in (79). In (79a), the $\mathrm{IE}_{\mathbf{A}}$ alternative $K\forall$ may be pruned in contexts where it is not equivalent to $K\exists$, that is, where the background permits a difference between the truth values of these two propositions. These include contexts where it is not known whether every one of our students smokes, say, or where it is known that not all of them do. It is this possibility that allows suppression of the $\forall$ inference in (79a). Therefore the $\forall$ inference is optional, as is its negation. Note that the $\neg\forall$ inference may be derived from embedded exhaustification in this case, as originally argued by Chierchia (2004). In (79b), on the other hand, the $\neg K\forall$ alternative may only be pruned from $\mathrm{IE}_{\mathbf{Pr}}$ when its presupposition is not contextually equivalent to that of the original sentence. But this requires a background that does not support $\forall$; any context that supports $\forall$ makes the presuppositions of (79b) and its $\mathrm{IE}_{\mathbf{Pr}}$ alternative equivalent, and in such a background, (87) forces $\neg K\forall$ to stay in the $\mathrm{IE}_{\mathbf{Pr}}$ and consequently to produce the $\neg\forall$ inference. This contradicts the background of the context. It follows that (79b) cannot be uttered in $\forall$ contexts.

The remaining updates to DOM-EXH involve factoring in the notion of Innocent Includability. II-alternatives on this revised view are divided, like IE-alternatives, into presuppositional and assertoric ones. The former must be selected from Pr; they must be Strawson-entailed by the prejacent. The latter are selected from the complement of Pr. The basis of the selection is the familiar condition of Innocent Includability, repeated below from Section 2.1.

(88) $$\begin{aligned}\mathrm{II}(C)(\phi) = \{\psi : \psi \in C \;\&\; & \forall B(B \subseteq C \;\&\; (\textstyle\bigwedge B \wedge \phi \wedge IE^{\neg}_{\phi,C} \nvDash \bot) \\ & \quad \rightarrow (\textstyle\bigwedge B \wedge \phi \wedge IE^{\neg}_{\phi,C} \wedge \psi \nvDash \bot))\}\end{aligned}$$

Our updates to DOM-EXH will now involve adding to $Dom(\mathrm{Exh}(C)(\phi))$ the presuppositions of its $\mathrm{II}_{\mathbf{Pr}}$ alternatives as well as those of its $\mathrm{II}_{\mathbf{A}}$ alternatives. These two sets are defined as follows: the $\mathrm{II}_{\mathbf{Pr}}$ alternatives are those elements of $\mathrm{Pr}(C)(\phi)$ whose presuppositions may be added consistently (innocently), given the presuppositions of the other members of $\mathrm{Pr}(C)(\phi)$ and given ϕ. So the addition of the $\mathrm{II}_{\mathbf{Pr}}$ presuppositions is effectively an II-enrichment that takes place in the presuppositional tier.

(89) $$\mathrm{II}_{\mathbf{Pr}}(C)(\phi) = \{\psi : \psi \in \mathrm{Pr}(C)(\phi) \;\&\; Dom(\psi) \in \mathrm{II}(\mathrm{Pr}^{Dom}(C)(\phi))(\phi)\}$$

Finally we define $\mathrm{II}_{\mathbf{A}}(C)(\phi)$: the alternatives that are assertorically II. These must come from outside of $\mathrm{Pr}(C)(\phi)$, and must have presuppositions and assertions that can together

[32] Magri uses (87a) to explain the obligatoriness of implicatures in sentences like *Some Italians come from a warm country*. Such sentences are odd. Since world knowledge tells us that the sentence is equivalent to its *all*-alternative, it follows from (87a) that the *all*-alternative cannot be pruned, and that the (false) "not all" implicature is obligatory. To Magri, this explains the oddness of the sentence.

be added without contradiction to ϕ, after its enrichment with the $\mathrm{II}_{\mathbf{Pr}}$/$\mathrm{IE}_{\mathbf{Pr}}$-presuppositions and the $\mathrm{IE}_{\mathbf{A}}$-assertions. The last of these conditions is encoded in the definition of innocent includability, so we need not repeat it. We write instead that an alternative ψ is $\mathrm{II}_{\mathbf{A}}$ w.r.t. C,ϕ iff (a) ψ is not a presuppositional alternative, and (b) among non-presuppositional alternatives, ψ is II with respect to $\phi_{+\mathrm{IE}_{\mathbf{Pr}}+\mathrm{II}_{\mathbf{Pr}}(C)}$. This proposition is none other than ϕ, but defined only in worlds where the presuppositions of the $\mathrm{IE}_{\mathbf{Pr}}$ alternatives are false and where those of its $\mathrm{II}_{\mathbf{Pr}}$ alternatives are true.

(90) $\phi_{+\mathrm{IE}_{\mathbf{Pr}}+\mathrm{II}_{\mathbf{Pr}}(C)} = [\lambda w : w \in (Dom(\phi) \cap \bigcap \mathrm{II}^{Dom}_{\mathbf{Pr}}(C)(\phi) - \bigcup \mathrm{IE}^{Dom}_{\mathbf{Pr}}(C)(\phi))\,.\,\phi(w)]$

(91) $\mathrm{II}_{\mathbf{A}}(C)(\phi) = \mathrm{II}(C - \mathrm{Pr}(C)(\phi))(\phi_{+\mathrm{IE}_{\mathbf{Pr}}+\mathrm{II}_{\mathbf{Pr}}(C)})$

Now we come to our final version of DOM-EXH: exhaustification of ϕ, given C, brings with it the presuppositions of ϕ's $\mathrm{IE}_{\mathbf{A}}$/$\mathrm{II}_{\mathbf{A}}$ alternatives, the presuppositions of its $\mathrm{II}_{\mathbf{Pr}}$ alternatives, and the negations of the presuppositions of its $\mathrm{IE}_{\mathbf{Pr}}$ alternatives.

(92) **Definedness condition on Exh (Take 5) – DOM-EXH5**:

$$Dom(\mathrm{Exh}(C)(\phi)) \subseteq Dom(\phi) \cap \bigcap \mathrm{II}^{Dom}_{\mathbf{Pr}}(C)(\phi) \cap \bigcap \mathrm{IE}^{Dom}_{\mathbf{A}}(C)(\phi) \cap \bigcap \mathrm{II}^{Dom}_{\mathbf{A}}(C)(\phi) - \bigcup \mathrm{IE}^{Dom}_{\mathbf{Pr}}(C)(\phi)$$

In the rest of this section I will use the following examples to show how DOM-EXH works. I note ahead of time that I will not show examples where assertoric II-alternatives play a role, since those do not come up in the cases that interest us. We will look at *only* and disjunction in Section 4.3.

(93) Kim is unaware that Mel ate soup or salad. $\neg K(p \vee q)$
Target presupposition: Mel did not eat both soup and salad. $\neg(p \wedge q)$

(94) Kim is unaware that Mel is required to eat soup or salad. $\neg K\Box(p \vee q)$
Target presupposition: Mel is not required to eat soup, $\neg\Box p$
Mel is not required to eat salad. $\neg\Box q$

(95) Kim is unaware that Mel is allowed to eat soup or salad. $\neg K\Diamond(p \vee q)$
Target presupposition: Mel is not allowed to eat soup and salad, $\neg\Diamond(p \wedge q)$
Mel is allowed to eat soup, $\Diamond p$
Mel is allowed to eat salad. $\Diamond q$

Begin with (93). Here, as indeed in all of (93)-(95), alternatives where disjunction is replaced with the disjuncts and with the conjunction are Strawson-entailed by the sentence. But because in each case the replacement makes a stronger presupposition, it follows that in all of them the alternatives of interest are presuppositional. In the case of (93), only the conjunctive alternative qualifies as $\mathrm{IE}_{\mathbf{Pr}}$:

(93′) Let $\phi = \neg K(p \vee q)$ and $C = \{\neg Kp, \neg Kq, \neg K(p \wedge q)\}$. Then:

a. $\mathrm{Pr}(C)(\phi) = \{\neg Kp, \neg Kq, \neg K(p \wedge q)\}$
b. $\mathrm{Pr}^{Dom}(C)(\phi) = \{p, q, (p \wedge q)\}$
c. $\mathrm{IE}_{\mathbf{Pr}}(C)(\phi) = \{\neg K(p \wedge q)\}$
d. $\mathrm{IE}^{Dom}_{\mathbf{Pr}}(C)(\phi) = \{(p \wedge q)\}$

e. $\text{IE}_{\mathbf{A}}(C)(\phi) = \{\}$

Note also that no presuppositional alternative is innocently includable in this case, since the presuppositions of $\neg Kp$ and $\neg Kq$ — the propositions p, q — are not II given $\text{Pr}^{Dom}(C)(\phi) = \{p, q, (p \wedge q)\}$ and $Dom(\phi) = [p \vee q]$:

(93″) For $\phi = \neg K(p \vee q)$ and $C = \{\neg Kp, \neg Kq, \neg K(p \wedge q)\}$,

a. $\text{II}(\text{Pr}^{Dom}(C)(\phi))(\phi) = \text{II}(\{p, q, p \wedge q\})(\neg K(p \vee q)) = \{\}$

b. $\text{II}_{\mathbf{Pr}}(C)(\phi) = \{\}$

Therefore by DOM-EXH we predict that (93) presuppose that Mel ate soup or salad but did not eat both, as desired.[33] The irrelevant details from DOM-EXH are greyed out.

(93‴) For $\phi = \neg K(p \vee q)$ and $C = \{\neg Kp, \neg Kq, \neg K(p \wedge q)\}$,

$$\begin{aligned} Dom(\text{Exh}(C)(\phi)) &\subseteq Dom(\phi) \cap \bigcap \text{II}^{Dom}_{\mathbf{Pr}}(C)(\phi) \cap \bigcap \text{IE}^{Dom}_{\mathbf{A}}(C)(\phi) \\ &\quad \cap \bigcap \text{II}^{Dom}_{\mathbf{A}}(C)(\phi) - \bigcup \text{IE}^{Dom}_{\mathbf{Pr}}(C)(\phi), \text{ i.e.} \\ &\subseteq (p \vee q) - \bigcup \{(p \wedge q)\} \end{aligned}$$

Things are slightly different in (94). Here, all of the alternatives of interest are (presuppositionally) IE, not just the conjunctive alternative. For this reason, the sentence is predicted to presuppose that Mel is required to eat soup or salad, but not required to eat either one specifically. This is also as desired.

(94′) Let $\phi = \neg K\Box(p \vee q)$ and $C = \{\neg K\Box p, \neg K\Box q, \neg K\Box(p \wedge q)\}$. Then:

a. $\text{Pr}(C)(\phi) = \{\neg K\Box p, \neg K\Box q, \neg K\Box(p \wedge q)\}$

b. $\text{Pr}^{Dom}(C)(\phi) = \{\Box p, \Box q, \Box(p \wedge q)\}$

c. $\text{IE}_{\mathbf{Pr}}(C)(\phi) = \{\neg K\Box p, \neg K\Box q, \neg K\Box(p \wedge q)\}$

d. $\text{IE}^{Dom}_{\mathbf{Pr}}(C)(\phi) = \{\Box p, \Box q, \Box(p \wedge q)\}$

e. $\text{IE}_{\mathbf{A}}(C)(\phi) = \{\}$

I leave it to the reader to confirm that $\text{Pr}(C)(\phi)$ has no $\text{II}_{\mathbf{Pr}}$ alternatives in this case. The final result is summarized below:

(94″) For $\phi = \neg K\Box(p \vee q)$ and $C = \{\neg K\Box p, \neg K\Box q, \neg\Box K(p \wedge q)\}$,

$$\begin{aligned} Dom(\text{Exh}(C)(\phi)) &\subseteq Dom(\phi) \cap \bigcap \text{II}^{Dom}_{\mathbf{Pr}}(C)(\phi) \cap \bigcap \text{IE}^{Dom}_{\mathbf{A}}(C)(\phi) \\ &\quad \cap \bigcap \text{II}^{Dom}_{\mathbf{A}}(C)(\phi) - \bigcup \text{IE}^{Dom}_{\mathbf{Pr}}(C)(\phi), \text{ i.e.} \\ &\subseteq \Box(p \vee q) - \bigcup \{\Box p, \Box q, \Box(p \wedge q)\} \end{aligned}$$

And in the case of (95), we derive the FC presupposition $\Diamond p, \Diamond q$ from the $\text{II}_{\mathbf{Pr}}$ alternatives. The presuppositional alternatives here are $\neg K\Diamond p$, $\neg K\Diamond q$, and $\neg K\Diamond(p \wedge q)$. These presuppose $\Diamond p$, $\Diamond q$, and $\Diamond(p \wedge q)$, and only the last of these propositions is innocently excludable against $\neg K\Diamond(p \vee q)$. That is:

(95′) Let $\phi = \neg K\Diamond(p \vee q)$ and $C = \{\neg K\Diamond p, \neg K\Diamond q, \neg K\Diamond(p \wedge q)\}$. Then:

[33] There is also a presupposition of ignorance in this case, which I do not discuss. See Spector and Sudo 2017 and Marty and Romoli 2020.

a. $\mathrm{Pr}(C)(\phi) = \{\neg K\Diamond p, \neg K\Diamond q, \neg K\Diamond(p \wedge q)\}$
b. $\mathrm{Pr}^{Dom}(C)(\phi) = \{\Diamond p, \Diamond q, \Diamond(p \wedge q)\}$
c. $\mathrm{IE}_{\mathbf{Pr}}(C)(\phi) = \{\neg K\Diamond(p \wedge q)\}$
d. $\mathrm{IE}^{Dom}_{\mathbf{Pr}}(C)(\phi) = \{\Diamond(p \wedge q)\}$
e. $\mathrm{IE}_{\mathbf{A}}(C)(\phi) = \{\}$

This time we find two presuppositional alternatives with II presuppositions: $\neg K\Diamond p$ and $\neg K\Diamond q$:

(95″) For $\phi = \neg K\Diamond(p \vee q)$ and $C = \{\neg K\Diamond p, \neg K\Diamond q, \neg K\Diamond(p \wedge q)\}$,
a. $\mathrm{II}(\mathrm{Pr}^{Dom}(C)(\phi))(\phi) = \mathrm{II}(\{\Diamond p, \Diamond q, \Diamond(p \wedge q)\})(\neg K\Diamond(p \vee q)) = \{\Diamond p, \Diamond q\}$
b. $\mathrm{II}_{\mathbf{Pr}}(C)(\phi) = \{\neg K\Diamond p, \neg K\Diamond q\}$
c. $\mathrm{II}^{Dom}_{\mathbf{Pr}}(C)(\phi) = \{\Diamond p, \Diamond q\}$

And with DOM-EXH, we add to (95) the presupposition that Mel is not permitted to eat soup and salad, and the presupposition that s/he is permitted to eat soup and permitted to eat salad:

(95‴) For $\phi = \neg K\Diamond(p \vee q)$ and $C = \{\neg K\Diamond p, \neg K\Diamond q, \neg\Diamond K(p \wedge q)\}$,

$$\begin{aligned} Dom(\mathrm{Exh}(C)(\phi)) &\subseteq Dom(\phi) \cap \bigcap \mathrm{II}^{Dom}_{\mathbf{Pr}}(C)(\phi) \cap \bigcap \mathrm{IE}^{Dom}_{\mathbf{A}}(C)(\phi) \\ &\quad \cap \bigcap \mathrm{II}^{Dom}_{\mathbf{A}}(C)(\phi) - \bigcup \mathrm{IE}^{Dom}_{\mathbf{Pr}}(C)(\phi), \text{ i.e.} \\ &\subseteq \Diamond(p \vee q) \cap \bigcap\{\Diamond p, \Diamond q\} - \bigcup\{\Diamond(p \wedge q)\} \end{aligned}$$

In the next section I show how DOM-EXH, formulated within Marty and Romoli's view, produces the inferences we want when *only* takes a disjunctive prejacent.

4.3 Application to *only* and disjunctive prejacents

Much of what I will show in this section repeats results that were derived earlier. So to save space, I want to take note of (and use) a connection between the simple version of DOM-EXH developed in Section 4.1 and the more elaborate version that follows Marty and Romoli.

Recall that one of the motivations for adopting the second account comes from a difference between the presuppositional implicatures of *know* and those of *unaware*. Ultimately, the difference depended on positing an assertoric set of alternatives and a presuppositional set. What I want to note is this: if we limit our attention to just the presuppositional alternatives, we find that the two accounts produce the same result. This claim, if it is accurate, tells us that the presuppositional implicatures of *unaware*, whose alternatives of interest were all presuppositional, are predicted to be the same regardless of whether we use the simple view of DOM-EXH or the version based on Marty and Romoli. And because *only* is similarly Strawson-downward entailing, we expect that the results we got on the simple view (for *only* and disjunctive prejacents) should be the same on the updated view.

Recall that the basic idea on the simple view is that the presupposition of the prejacent ϕ is exhaustified relative to the presuppositions of its alternatives. This was made precise by writing DOM-EXH like this:

(96) $Dom(\text{Exh}(C)(\phi)) \subseteq \text{Exh}(C^{Dom})(Dom(\phi))$

Now, imagine that instead of C, we have only $\text{Pr}(C)(\phi)$, ϕ's presuppositional alternatives. Then (96) would be written as follows,

(97) $Dom(\text{Exh}(\text{Pr}(C)(\phi))(\phi)) \subseteq \text{Exh}(\text{Pr}^{Dom}(C)(\phi))(Dom(\phi))$

What is the result of applying Exh to $\text{Pr}^{Dom}(C)(\phi)$ and $Dom(\phi)$? The answer is a conjunction of three things: (i) the propositional argument, here $Dom(\phi)$, (ii) the negations of whatever is IE from $\text{Pr}^{Dom}(C)(\phi)$, and (iii) the assertions of whatever is II from $\text{Pr}^{Dom}(C)(\phi)$. So (97) may be rewritten like this:

(98) $Dom(\text{Exh}(\text{Pr}(C)(\phi))(\phi)) \subseteq [Dom(\phi) \wedge II_{Dom(\phi),\text{Pr}^{Dom}(C)(\phi)} \wedge IE^{\neg}_{Dom(\phi),\text{Pr}^{Dom}(C)(\phi)}]$

On the other hand, M&R's account restricts the domain of $\text{Exh}(C)(\phi)$ as follows:

(99) $Dom(\text{Exh}(C)(\phi)) \subseteq Dom(\phi) \cap \bigcap \text{II}^{Dom}_{\mathbf{Pr}}(C)(\phi) \cap \bigcap \text{IE}^{Dom}_{\mathbf{A}}(C)(\phi)$
$\cap \bigcap \text{II}^{Dom}_{\mathbf{A}}(C)(\phi) - \bigcup \text{IE}^{Dom}_{\mathbf{Pr}}(C)(\phi)$

Since we are interested in cases where there are no assertoric alternatives, we can drop $\text{IE}_{\mathbf{A}}$ and $\text{II}_{\mathbf{A}}$ from (99). This gives us (100):

(100) $Dom(\text{Exh}(\text{Pr}(C)(\phi))(\phi)) \subseteq Dom(\phi) \cap \bigcap \text{II}^{Dom}_{\mathbf{Pr}}(C)(\phi) - \bigcup \text{IE}^{Dom}_{\mathbf{Pr}}(C)(\phi)$

Now if we compare the right sides of (98) and (100), we find them to be very similar: the two propositions that are conjoined to $Dom(\phi)$ in (98) are, first, the conjunction of whatever presuppositions are II — given $Dom(\phi)$ — of the elements of $\text{Pr}(C)(\phi)$, and second, the conjunction of the negations of whatever presuppositions are IE — again given $Dom(\phi)$ — of the elements of $\text{Pr}(C)(\phi)$. The first of these shows up in (100) as the grand conjunction of $\text{II}^{Dom}_{\mathbf{Pr}}(C)(\phi)$, which contains the *presuppositions* that are II from the elements of $\text{Pr}(C)(\phi)$ given ϕ. Notice that here there is a small difference: II-ness is evaluated relative to ϕ, while in (98) it is relative to $Dom(\phi)$. I do not know if there are cases where this distinguishes the predictions of the two accounts. For our data specifically, there is no difference. The same point holds of the *IE* conjunct in (98) and its parallel in (100). The conjunction of negations in the first is equivalent to the negation of disjunctions (subtraction of the grand union) in the second.

So the results we obtained from applying the simple account to *only* and its disjunctive prejacents are predicted to be the same here, as long as the key formal alternatives can all be shown to be presuppositional. This is not a trivial matter, it turns out, so in the rest of this section I will talk about what we need to assume to produce the correct outcome.

To begin, we must recall the two assumptions from Section 4.1. First, when *only* has a disjunctive focus-associate, the particle does not operate on alternatives where the disjuncts replace that associate. Second, alternatives whose presuppositions are incompatible with Exh's prejacent are pruned from the set of alternatives. Recall (63):

(63) Kim only used [a teaspoon]$_{\text{F}}$ of vinegar.

On the simple definition of DOM-EXH, (63) should presuppose the negation of the (stronger) presupposition of (64). This, however, is incorrect, since the inference is intuitively part of *only*'s meaning.

(64) Kim only used [a tablespoon]$_{\mathrm{F}}$ of vinegar.

When this problem was pointed out in the discussion of the simple view, the solution to it was to prune alternatives like (64) from the alternatives that feed Exh. The move was justified on the grounds that such alternatives conflict with Exh's prejacent — the prejacent already entails that their presuppositions are false — so limiting Exh's attention away from them makes Exh more economical.

In the context of M&R, the same problem comes up; by the formulation of $\mathrm{IE}_{\mathbf{Pr}}$, (63) is predicted to have (64) as a presuppositional alternative, because (63) Strawson-entails (64) without logically entailing it. In this case the Strawson-entailment is trivial; there are no worlds where (63) is true and where the presupposition of (64) holds — such a world would have to be one where Kim did not use more than a teaspoon of vinegar, but used at least a tablespoon of it. This contradictory proposition vacuously entails (64), so (64) is a Strawson-consequence of (63). Once again, then, we get the unwanted prediction that (63) presuppose (upon exhaustification) that the presupposition of (64) is false. (64) must therefore be pruned from C on M&R's account, like it was on the simple account.

The point so far is that the assumptions we needed when we applied the simple account to *only* are also needed if we adopt M&R's view. There is, however, a new assumption that we need to add if we adopt M&R's view. It concerns cases where the disjunctive phrase is *only*'s focus associate, like (101), repeated from (1).

(101) Kim is only allowed to eat [soup or salad]$_{\mathrm{F}}$. $\quad only_C \Diamond(p \vee q)_{\mathrm{F}}$
Target presupposition: Kim is allowed to eat soup, $\quad \Diamond p$
Kim is allowed to eat salad. $\quad \Diamond q$

The formal alternatives to (101) appear in (102), also repeated:

(102) {Kim is only allowed to eat soup$_{\mathrm{F}}$, $\quad only(\Diamond p_{\mathrm{F}})$
Kim is only allowed to eat salad$_{\mathrm{F}}$, $\quad only(\Diamond q_{\mathrm{F}})$
Kim is only allowed to eat rice$_{\mathrm{F}}$, $\quad only(\Diamond r_{\mathrm{F}})$
Kim is only allowed to eat [soup and salad]$_{\mathrm{F}}, \cdots$} $\quad only(\Diamond(p \wedge q)_{\mathrm{F}})$

As before, the bottom two alternatives here conflict with the meaning of (101), so they are pruned from C. But consider the top two alternatives. Are they Strawson-entailed by (101)? The answer depends on what *only* asserts in them. If in the first alternative in (102) *only* says that Kim is not allowed to eat salad (only soup), then the alternative is not Strawson-entailed by (101): (101) says that Kim may eat at least one of soup and salad, but may not eat both and may not eat other things, and the presupposition of the alternative $only(\Diamond p_{\mathrm{F}})$ says that Kim may eat soup. These two premises do not lead to the conclusion that Kim may not eat salad, so the alternative does not Strawson-follow from (101). The same point holds of the second alternative.

It follows then, that the crucial alternatives in this case will not qualify as presuppositional, so their behavior under M&R's view will not be the same as it was on the simple view. The reason, of course, is that on the simple view only their *presuppositions* were considered for DOM-EXH; their status as Strawson-consequences of (101) did not matter.

Despite this apparent divergence between the two accounts, I want to suggest a way of making their predictions converge again. The alternatives in (102) are alternatives of (101), and in (101) the disjuncts $\Diamond p, \Diamond q$ are absent from *only*'s C (recall the constraints (51)-(52) on *only*'s C from the beginning of Sect. 4). If so, then there is an argument to be made for their absence from the occurrences of C in the elements of (102) also.[34] This would make the top two alternatives in (102) Strawson-consequences of (101) again, and would return them to the set of presuppositional alternatives.

5 Summary and conclusions

I have claimed that the FC presupposition that comes from embedding disjunctions under *only* results from enriching *only*'s prejacent presupposition with its "implicatures". I put this proposal in contrast to Bar-Lev and Fox's claim that in cases like (1), the FC presupposition comes directly from *only* — its II presupposition specifically. In my argument I showed evidence of FC presuppositions under *only* that are not captured by B-L&F's proposal, and argued further that disjuncts are not visible to *only* as formal alternatives when its associate is a disjunctive phrase. There is therefore no support for the hypothesis that *only* has an II component.

In fact, if my conclusion is right about how disjunctive focus-associates interact with *only*, it follows that there is no support for the idea that *only* has an IE component either, at least not on the basis of these examples. Everything that was derived in Section 4 can be derived if we revert to Krifka's (1993) entry for *only* (see (3)). As I said earlier, however, removing these details from *only*'s meaning does not justify a similar move in our theory of implicature calculation.

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[34] Fox (2007) made the same assumption about alternative-sets in his derivation of FC from recursive exhaustification. In $\text{Exh}_C(\text{Exh}_{C'}(\Diamond(p \vee q)))$, C' contains the disjunctive alternatives $\Diamond p, \Diamond q$, and C contains the pre-exhaustified alternatives $\text{Exh}_{C'}(\Diamond p)$ and $\text{Exh}_{C'}(\Diamond q)$. Crucially, C' in these two alternatives is the same as it is in $\text{Exh}_{C'}(\Diamond(p \vee q))$. This makes $\text{Exh}_{C'}(\Diamond p)$ mean $(\Diamond p \,\&\, \neg\Diamond q)$, and $\text{Exh}_{C'}(\Diamond q)$ mean $(\Diamond q \,\&\, \neg\Diamond p)$. When these alternatives are negated by the higher Exh, we get the desired inference $(\Diamond p \leftrightarrow \Diamond q)$, which entails FC when combined with the prejacent $\Diamond(p \vee q)$.

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