Free Choice Disjunctions under *only*[*]

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This paper concerns Free Choice[1] permission sentences under *only*. The Free Choice inference, FC from now on, is exemplified in (1).

(1)   You are allowed to eat cake or ice cream.   ($\Diamond(p \vee q)$ hereafter)

    a. $\vDash_{FC}$ You are allowed to eat cake.   ($\Diamond p$)
    b. $\vDash_{FC}$ You are allowed to eat ice cream.   ($\Diamond q$)

The challenge presented by (1), at least to traditional approaches to natural language semantics, is that neither of (1-a,b) seems to follow from the literal content of the sentence; if we take sentences to denote propositions (sets of possible worlds), then (1) holds in w iff some w' is compatible with the regulations in w, and in w' the addressee eats cake or eats ice cream. Since this disjunctive requirement is met when only one of the disjuncts is true, e.g. if the addressee eats just cake in w', we fail to derive the stronger inference licensed by (1), namely that eating cake is permitted *and* that eating ice cream is permitted.[2]

In more recent literature, beginning in Kratzer and Shimoyama (2002), the FC inference is argued to result from an extra-semantic component, reminiscent in its behavior of the mechanism with which Scalar Implicatures (SIs) are derived. This view is motivated by the finding that, when a sentence like (1) is negated, it is its weaker *disjunctive* content that is felt to be negated, and not the stronger (conjunctive) content of the FC inference.

(2)   You are not allowed to have cake or ice cream.
    $= \neg\Diamond(p \vee q)$
    $\neq \neg(\Diamond p \wedge \Diamond q)$

The disappearance of FC under negation (in DE environments generally) follows a trend

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[1]Kamp (1973).

[2]There are theories of natural language semantics that derive FC from the literal content of (1). See for example Aloni and Ciardelli (2013) for an inquisitive semantic account of FC, and Schulz (2007) for an account that uses minimal models.

that Scalar Implicatures make familiar, since they too are cancelled in DE. This similarity suggests the SI-based treatment proposed in Kratzer and Shimoyama, and elaborated in Alonso-Ovalle (2006), Fox (2007), among others.[3]

In this paper I focus on Fox's account of FC, and discuss how it might apply to sentences like (3), where (1) appears in the scope of *only*.

(3)   You are only allowed to eat [cake or ice cream]$_F$.

I will show that recursive exhaustification, the mechanism responsible for FC on Fox's account, does not carry over straightforwardly to (3). Specifically, I show that (3) differs from (1) in that it licenses three different types of inference. I explore ways of deriving the inferences by investigating the interaction between Exh and the presupposition of *only*, arriving finally at two ways of parsing (3) that each deliver the desired results.

1.   Background: FC by recursive exhaustification

On Fox's (2007) theory of FC, a covert operator Exh, similar in its semantics to *only*, is applied twice at the matrix level to sentences like (1), repeated.[4]

(1)   You are allowed to eat cake or ice cream. $(\Diamond(p \vee q))$

I assume that Exh applies to a sentential argument—its prejacent—and operates on its prejacent's alternatives. The alternatives to a given sentence S are determined by replacing the scalar items within S with their formal/contextually-supplied alternatives. Given a prejacent S and a set of alternatives A, $\llbracket \text{Exh}_A\, S \rrbracket$ is true in a world w iff (i) $\llbracket S \rrbracket$ is true in w, and (ii) every alternative in A which is Innocently-Excludable (IE) given S is false in w. This is summarized in (4) (IE will be defined shortly).

(4) $\llbracket \text{Exh}_A\, S \rrbracket = \lambda w\,.\, \llbracket S \rrbracket(w){=}1\, \&\, \bigwedge\{\llbracket S' \rrbracket(w){=}0 : S' \in \text{IE}(S)(A)\}$[5]

I assume also that a disjunction $p \vee q$ has its disjuncts p, q as alternatives, as well as their conjunction $p \wedge q$.[6] Let us now abbreviate (1) as $\Diamond(p \vee q)$, where p is '*you eat cake*', and q '*you eat ice cream*'. Since the embedded disjunction has its disjuncts (and their conjunction) as alternatives, we derive

(5) $A = \text{ALT}(\Diamond(p \vee q)) = \{\Diamond p, \Diamond q, \Diamond(p \wedge q)\}$

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[3]See also Franke (2011) for a game-theoretic account of FC.

[4]The exhaustification operator Exh plays a central role in the so-called 'grammatical' account of SIs. Advocates of grammatical theories of implicatures cite the presence of embedded SIs as an argument (among others) against the traditional/neo-Gricean views of SIs (e.g. the formulations in Gazdar 1979, Gamut 1991, and Sauerland 2004), and in favor of an account in which the relevant inferences are derived with Exh (e.g. Groenendijk and Stokhof 1984, Chierchia et al. 2012, Sauerland 2012, and Fox 2014). For a review of the Neo-Gricean/grammatical debate, see Geurts (2010) and Schlenker (to appear).

[5] Note that IE-alternatives need to be *false* in this formulation. This will be discussed in Section 3 when we investigate the interaction between implicatures and presuppositions.

[6]See Sauerland (2004) and Katzir (2007) for more on alternatives.

Not all of the alternatives in *A* will qualify as Innocently-Excludable (IE) with respect to the prejacent $\Diamond(p \vee q)$. IE alternatives are those that are left in *A* after we subtract from it every subset *B* that *minimally-covers* the prejacent.[7] We say that a set of sentences *B* *covers* a sentence *S* iff the negations of *B*'s elements are jointly inconsistent with *S*. We say that *B* *minimally-covers S* whenever *B* covers *S*, and no proper subset of *B* covers *S* (we will see an example shortly):

(6)   Let $B^{\text{neg}} = \{\phi : \exists S' \in B$ and $\phi = [\lambda w.[\![S']\!](w) = 0]\}$. Then, given a sentence *S* and a set of sentences *B*,

    a.   *B* <u>covers</u> *S* iff $[\![S]\!] \wedge \bigwedge B^{\text{neg}} \models \bot$.

    b.   *B* <u>minimally-covers</u> *S* iff *B* covers *S*, and no proper subset of *B* covers *S*.

Now, given $S = \Diamond(p \vee q)$ and $A = (5)$, it can be seen that *A* itself covers *S*, since the negations of its elements—$\Diamond p$, $\Diamond q$, and $\Diamond(p \wedge q)$—are jointly inconsistent with $\Diamond(p \vee q)$. However, *A* does not *minimally*-cover *S*, because there is a proper subset of it, namely $B = \{\Diamond p, \Diamond q\}$, which also covers *S*; the negation of $\Diamond p$ and the negation of $\Diamond q$ jointly contradict *S*. And since no proper subset of *B* covers *S*, *B* can be said to minimally-cover *S*. (Note also that no other subset of *A* covers *S*).

We return now to Innocent-Exclusion. We said that, given *S* and set *A*, the set of IE-alternatives is the result of subtracting from *A* all sets *B* that minimally-cover *S*.

(7) $\text{IE}(S)(A) = A - \cup \{B : B \text{ minimally-covers } S\}$

In our example, we found that $\{\Diamond p, \Diamond q\}$ minimally-covers $S = \Diamond(p \vee q)$. This, combined with the definition of IE in (7), gives us (8), and as a result (9).

(8) $\text{IE}(\Diamond(p \vee q))(A) = A - \cup\{\{\Diamond p, \Diamond q\}\}$

            $= \{\Diamond p, \Diamond q, \Diamond(p \wedge q)\} - \{\Diamond p, \Diamond q\} = \{\Diamond(p \wedge q)\}$

(9) $[\![\text{Exh}_A \Diamond(p \vee q)]\!] = \lambda w . [\![\Diamond(p \vee q)]\!](w) = 1 \& \bigwedge\{[\![S']\!](w) = 0 : S' \in (8)\}$

            $= \lambda w . [\![\Diamond(p \vee q)]\!](w) = 1 \& [\![\Diamond(p \wedge q)]\!](w) = 0$

Exhaustifying $S = \Diamond(p \vee q)$ given *A*, then, strengthens its denotation by adding to it the negation of the alternative $\Diamond(p \wedge q)$, i.e. the exclusive inference.

The FC is derived in Fox by applying Exh *recursively* to sentences like (1):

(10) $[\![\text{Exh}_{A'} (\text{Exh}_A \Diamond(p \vee q))]\!] \subseteq \lambda w . [\![\Diamond p]\!](w) = 1 \& [\![\Diamond q]\!](w) = 1$ (Fox 2007)

To see how (10) works, we need to determine the contents of *A* and *A'*, the alternatives sets of the inner Exh and the outer Exh, respectively. As in the earlier example, *A* is determined by replacing the scalar terms in the prejacent of Exh_A, i.e. $\Diamond(p \vee q)$, with their alternatives:

(5) $A = \text{ALT}(\Diamond(p \vee q)) = \{\Diamond p, \ \Diamond q, \ \Diamond(p \wedge q)\}$ (repeated)

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[7]The definition of IE is stated differently here from the way it is stated in Fox. The two definitions are equivalent as far as I can see.

The alternatives in A' are determined, likewise, by substituting the scalar terms in the pre-jacent of $\text{Exh}_{A'}$ with their alternatives. Here the prejacent, $S' = \text{Exh}_A \Diamond(p \vee q)$, contains an occurrence of Exh, and so its alternatives will each contain Exh_A, and will vary according to the formal alternatives of the disjunction appearing under Exh_A. This gives us (11).

(11) $A' = \text{ALT}(\text{Exh}_A \Diamond(p \vee q)) = \{\text{Exh}_A\Diamond p, \ \text{Exh}_A\Diamond q, \ \text{Exh}_A\Diamond(p \wedge q)\}$

It will now be shown that neither A' nor any of its subsets cover the prejacent S'—minimally or not—and that (consequently) nothing is subtracted from A' in deriving $\text{IE}(S')(A')$. The contents of A' denote, respectively, the propositions that p (cake) is allowed but q (ice cream) is not; that q is allowed but p is not; and that $(p \wedge q)$ is allowed.[8] Negating these propositions gives us, respectively, the proposition that if p is allowed then q is; that if q is allowed then p is, and that $(p \wedge q)$ is not allowed. These negations do *not* jointly contradict the prejacent, because there are possible worlds in which they, along with the prejacent, are all simultaneously satisfied: in worlds where p and q are individually permitted, but where $p \wedge q$ is not, the prejacent is true, and the negations of A' are also true.[9] This shows that the negations of A' are not inconsistent with S', which means that A' does not cover S'. And of course, if A' does not cover S', then (*a fortiori*) none of its subsets do.

(12) $\text{IE}(S')(A') = A' - \emptyset = \{\text{Exh}_A\Diamond p, \ \text{Exh}_A\Diamond q, \ \text{Exh}_A\Diamond(p \wedge q)\}$

The resulting denotation of $\text{Exh}_{A'}(\text{Exh}_A \Diamond(p \vee q))$ is derived in detail below (I strike out parts that follow from other parts of the same line. The latter are underlined with dots).

(13) $[\![\text{Exh}_{A'} \ S']\!] = \lambda w . [\![S']\!](w){=}1 \ \& \ \bigwedge\{[\![S'']\!](w){=}0 : S'' \in (12)\}$

   $= \lambda w . \boxed{\text{A}}(w){=}1 \ \& \ \boxed{\text{B}}(w){=}0 \ \& \ \boxed{\text{C}}(w){=}0 \ \& \ \boxed{\text{D}}(w){=}0$

   $= \lambda w . [\![\Diamond(p \vee q)]\!](w){=}1 \ \& \ \cancel{[\![\Diamond(p \wedge q)]\!](w){=}0}$

      $\& \ ([\![\Diamond p]\!](w){=}1 \to [\![\Diamond q]\!](w){=}1) \ \& \ ([\![\Diamond q]\!](w){=}1 \to [\![\Diamond p]\!](w){=}1)$

      $\& \ [\![\Diamond(p \wedge q)]\!](w){=}0$

   $= \lambda w . [\![\Diamond(p \vee q)]\!](w){=}1 \ \& \ ([\![\Diamond p]\!](w){=}1 \leftrightarrow [\![\Diamond q]\!](w){=}1)$

      $\& \ [\![\Diamond(p \wedge q)]\!](w){=}0$

   $\boxed{= \lambda w . [\![\Diamond p]\!](w){=}1 \ \& \ [\![\Diamond q]\!](w){=}1 \ \& \ [\![\Diamond(p \wedge q)]\!](w){=}0}$

$\boxed{\text{A}}$ $[\![\text{Exh}_A \ \Diamond(p \vee q)]\!] = \lambda w . [\![\Diamond(p \vee q)]\!](w){=}1 \ \& \ \bigwedge\{[\![S']\!](w){=}0 : S' \in (8)\}$

   $= \lambda w . [\![\Diamond(p \vee q)]\!](w){=}1 \ \& \ [\![\Diamond(p \wedge q)]\!](w){=}0$ $(=(9))$

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[8] These denotations result from applying Exh_A to the embedded alternatives $\Diamond p$, $\Diamond q$, and $\Diamond(p \wedge q)$. Note that in the latter case, Exh_A does not strengthen the meaning of its prejacent at all, since $\text{IE}(\Diamond(p \wedge q))(A)$ is empty, as the reader may verify (see footnote 10).

[9] If p and q are individually permitted, then the prejacent $S' = \text{Exh}_A\Diamond(p \vee q)$ is true, and it is true that if $\Diamond p$ then $\Diamond q$, and vice versa (the negations of $\text{Exh}_A\Diamond p$ and $\text{Exh}_A\Diamond q$, respectively), and finally, it is true that $(p \wedge q)$ is not permitted (the negation of $\Diamond(p \wedge q)$). The negations of A' are therefore consistent with the prejacent.

$\boxed{\text{B}}$ $[\![\text{Exh}_A \lozenge p]\!] = \lambda w.[\![\lozenge p]\!](w) {=} 1 \ \& \ \bigwedge\{[\![S']\!](w) {=} 0 : S' \in \{\lozenge q, \lozenge(p \wedge q)\}\}$

$= \lambda w.[\![\lozenge p]\!](w) {=} 1 \ \& \ [\![\lozenge q]\!](w) {=} 0 \ \&\,[\![\lozenge(p\wedge q)]\!](w){=}0$

$\boxed{\text{C}}$ $[\![\text{Exh}_A \lozenge q]\!] = \lambda w.[\![\lozenge q]\!](w) {=} 1 \ \& \ \bigwedge\{[\![S']\!](w) {=} 0 : S' \in \{\lozenge p, \lozenge(p \wedge q)\}\}$

$= \lambda w.[\![\lozenge q]\!](w) {=} 1 \ \& \ [\![\lozenge p]\!](w) {=} 0 \ \&\,[\![\lozenge(p\wedge q)]\!](w){=}0$

$\boxed{\text{D}}$ $[\![\text{Exh}_A \lozenge(p \wedge q)]\!] = \lambda w.[\![\lozenge(p \wedge q)]\!](w) {=} 1 \ \& \ \bigwedge\{[\![S']\!](w) {=} 0 : S' \in \emptyset\}$

$= \lambda w.[\![\lozenge(p \wedge q)]\!](w) {=} 1$

As is evident from (13), both FC and the exclusive inference $[\![\lozenge(p \wedge q)]\!] {=} 0$ result from exhaustification on Fox's account. We might note, moreover, that the former can be derived independently of the latter: if $\lozenge(p \wedge q)$ is absent from A, and $\text{Exh}_A\lozenge(p \wedge q)$ is absent from A', then we can repeat the derivation of FC in (13), but this time we would leave out (i) the negation of $[\![\lozenge(p \wedge q)]\!]$ in $\boxed{\text{A}}$, (ii) the struck-out material in $\boxed{\text{B}}$, $\boxed{\text{C}}$, and (iii) the entirety of $\boxed{\text{D}}$. This would generate FC without the exclusive inference.[10]

An important final note concerns the addition of other alternatives, e.g. contextually-salient ones. Suppose eating cookies (call it r) is taken into consideration (in addition to our p and q). Then the account generates the prohibition against r also from exhaustification; if $A = \{\lozenge p, \lozenge q, \lozenge r, \lozenge(p \wedge q)\}$ and $A' = \{\text{Exh}_A\lozenge p, \text{Exh}_A\lozenge q, \text{Exh}_A\lozenge r, \text{Exh}_A\lozenge(p \wedge q)\}$, then

(14) $[\![\text{Exh}_{A'}(\text{Exh}_A\lozenge(p \vee q))]\!] = \lambda w.[\![\lozenge p]\!](w){=}1 \ \& \ [\![\lozenge q]\!](w){=}1$ (FC)

$\& \ [\![\lozenge(p \wedge q)]\!](w){=}0$ (exclusive inference)

$\& \ [\![\lozenge r]\!](w){=}0$ (exclusion of $\lozenge r$)

In the next section, when I apply Fox's account to *only*, I will show that the three inferences in (14) behave differently. Specifically, I will show that, under *only*,

(i)   the independent alternative is negated semantically, by *only*,

(ii)   the FC inference behaves like a presupposition, and

(iii) the exclusive inference behaves like an implicature.

2.   Three types of inference under *only*

The central example of this paper is (15), uttered, without pitch-accent on *or*, in a context in which three types of dessert are salient: there is cake, ice cream, and cookies.[11]

(15)   You are only allowed to eat [cake or ice cream]$_\text{F}$.   (=(3))

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[10] Deriving FC without the conjunctive alternative makes the embedded Exh locally-vacuous, since, without $\lozenge(p \wedge q)$ as an alternative, $\text{Exh}\lozenge(p \vee q)$ will have the same truth conditions as $\lozenge(p \vee q)$. I will not assume that local vacuity prevents Exh from appearing at LF, however, especially given the fact that, though it is locally-vacuous, Exh can make non-trivial contributions globally, as in the current example.

[11] The analog of (15) where *or* is accented also licenses FC, which is interesting because neither $\lozenge p$ nor $\lozenge q$ is obviously an alternative to $\lozenge(p \text{ or}_\text{F} q)$, and $\lozenge p$, $\lozenge q$ are crucial in deriving FC from exhaustification. I leave this issue for future work.

The entry that I will assume for *only* is similar to that of Exh; the difference is that, while Exh asserts its prejacent, *only* presupposes it.[12] I follow the literature on *only* and call its two components the *presuppositional* and the *assertoric* (singly and doubly underlined below).

(16) $[\![only_A\ S]\!] = \lambda w : \underline{[\![S]\!](w)\!=\!1} \cdot \underline{\underline{\bigwedge\{[\![S']\!](w)\!=\!0 : S' \in \mathrm{IE}(S)(A)\}}}$

I assume also the same mechanism for generating alternatives, but in the case of *only* it is the focused-marked constituent that undergoes replacement. In (15), where [cake or ice cream] is focus-marked, we derive the same contents for *A* that we did under Exh.

The data.   First, I want to highlight the contrast between the exclusive inference in (15) and the negation of the independent alternative $\Diamond r$. Suppose A asks B, who is an authority, what s/he is allowed to eat. Consider the following two responses.

(17)   B: For sure you are allowed to have cake or ice cream . . .

> ✓Let me check if you can have both.
> ✓Let me check if you can have cookies.

(18)   B: For sure you are only allowed to have [cake or ice cream]$_F$ . . .

> ✓Let me check if you can have both.
> #Let me check if you can have cookies.

The contrast in (18) suggests that $\neg\Diamond r$, the prohibition against cookies, is contributed semantically by *only*'s assertoric component. This would explain the oddness of the second continuation. In contrast, the acceptability of the first continuation suggests that the exclusive inference is not part of the semantics of the sentence, which in turn means that $\neg\Diamond(p \wedge q)$ is not excluded by *only*. Compare this contrast to (17), where both inferences seem to be optional, given the acceptability of both of B's continuations.

Second, I argue that the FC inference in (15) behaves like a presupposition (rather than an SI). This distinguishes (15) from the *only*-less variant discussed earlier, where FC was derived by exhaustification. To see that FC is a presupposition under *only*, consider (19).

(19)   The map of the park shows five trails, some of which are not open to the public. I just asked a friend who is familiar with this park, and he doesn't think that we are only allowed to take trail [A or B]$_F$. #So if we take A, we might be fined.

In (19), the implied ban on taking trail A contradicts what is attributed earlier to the knowledgeable friend, i.e. that we are not only allowed to take trail A or B.[13] This contrasts with (20), where (semantic) negation succeeds in blocking FC, as it generally does for SIs (recall discussion of (2) on page 15).

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[12] This view goes back to Horn (1969). The inferential status of *only*'s prejacent's is not fully understood, however. See Ippolito (2008) and Beaver and Clark (2008) for reviews.

[13]I use the Neg-raising predicate *think* in (19), rather than explicit negation, in order to negate *only* semantically without using the *not only* collocation that some researchers believe is idiomatic (see e.g. Beaver and Clark (2008), Section 9.7).

(20)   ... I just asked a friend who is familiar with this park, and he doesn't think that we are allowed to take A or B. ✓ So if we take A, we might be fined.

Compare (19) also with (21), where the relevant permission sentence is replaced with one containing *and*.

(21)   ... I just asked a friend who is familiar with this park, and he doesn't think that we are allowed to take A and B. ✓ So if we take A, we might be fined.

In this case FC is blocked because the conjunctive sentence is negated, and since negation applies to the asserted conjunction, the rest of the discourse turns out consistent.

I take (19)-(21) to indicate that $only\Diamond(p \vee q)$ does not convey FC as an SI, nor as part of its assertoric meaning. If FC were an SI in (19), we would expect no difference between (19) and (20)—since as an SI, FC would be cancelled in both—and if FC were part of the assertoric meaning of $only\Diamond(p \vee q)$ we would expect no relevant difference between (19) and (21), for in both sentences negation applies to a conjunctive assertion, and should therefore be consistent with the speaker's continuation.

The fact that FC survives negation in (19) suggests that, under *only*, the inference behaves like a presupposition.[14] This finding will inform our later discussion of what possible parses sentence (15) can have, and how (assuming Fox's account of FC) Exh might interact with presuppositional items like *only*.

The third and final datum concerns the status of the exclusive inference. As we saw in (18), the prohibition against having both cake and ice cream was not felt to be as strong as the prohibition against cookies. Nonetheless, the exclusive inference does seem to be present, as suggested by the acceptability of the two continuations above, repeated in (22).

(22)   a.   For sure you are allowed to have cake or ice cream. Let me check if you can have both.
        b.   For sure you are only allowed to have [cake or ice cream]$_F$. Let me check if you can have both.

Judgements suggest that the exclusive inference is equally present in (22-a) and (22-b). So, if the inference is an implicature of one, it seems to be an implicature of the other. Note, moreover, that when either construction is embedded under the restrictor of a universal quantifier, an environment where SIs are usually cancelled, the inference is no longer detected. Assume again that three types of dessert are salient: cake, ice cream, and cookies.

(23)   a.   Guests who are allowed to eat cake or ice cream are lucky.
              ⊨ Guests who are allowed to eat both are lucky
        b.   Guests who are only allowed to eat [cake or ice cream]$_F$ are unlucky.
              ⊨ Guests who are allowed to eat both (but not cookies) are unlucky

I summarize these three empirical claims in (24).

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[14]It may be too strong to claim FC as a presupposition here, considering that it does not need be a piece of common knowledge in (19). I rather claim that FC has the same status as the prejacent of *only*, which some regard as a presupposition. (As mentioned in footnote 12, this issue is under debate).

(24)   In *only*$\Diamond(p \vee q)$,

    (i) $(\neg\Diamond r)$ is contributed by the assertoric component of *only*; $(\neg\Diamond(p \wedge q))$ is not.

    (ii) $(\Diamond p \wedge \Diamond q)$, FC, is a presupposition ($\approx$ has the same status as the prejacent).

    (iii) $(\neg\Diamond(p \wedge q))$ is an implicature ($\approx$ is contributed by Exh).

In the next section I discuss the theoretical implications of (24-i-iii).

3.   Exhaustification above/under *only*

In Section 1, it was shown how, on Fox's account, FC is derived from LFs that feature two Exh operators. We noted also that FC, together with the exclusive inference $\neg\Diamond(p \wedge q)$ and the negation of the independent alternative $\Diamond r$, are derived as SIs. In Section 2, I argued that under *only*, the three inferences behave differently. Now I turn to (15).

(15)   You are only allowed to eat [cake or ice cream]$_F$

The discussion will focus on how (15) might be parsed, and, given a particular parse, the assumptions that need to be made in order for the inferences to come out as desired. Before I turn to these questions, I want to point out that I will ignore the exclusive inference for the moment, and concern myself with FC and the exclusion of the independent alternative $\Diamond r$. Accordingly I will work with derivations in which the conjunctive alternative is absent, until I return to the exclusive inference in Section 3.1.

    Let us first take a doubly-exhaustified structure, and consider the possibility of replacing one of its two Exhs with *only*. This gives us the two parses in (25).

(25)   a. $only_{A'}(\text{Exh}_A\Diamond(p \vee q)_F)$

        b. $\text{Exh}_{A'}(only_A\Diamond(p \vee q)_F)$

(25-a), as shown in (30), incorrectly derives FC as part of the assertoric meaning of *only*, not as a presupposition. This conflicts with the empirical claims made in Section 2.[15] (In the final line of (30) I mark presuppositions in subscripts, a convention that I will use hereafter).

(26) $A = \text{ALT}(\Diamond(p \vee q)) = \{\Diamond p, \Diamond q, \Diamond r\}$;

(27) $\text{IE}(\Diamond(p \vee q))(A) = A - \{\Diamond p, \Diamond q\} = \{\Diamond r\}$

(28) $A' = \text{ALT}(\text{Exh}_A\Diamond(p \vee q)) = \{\text{Exh}_A\Diamond p, \text{Exh}_A\Diamond q, \text{Exh}_A\Diamond r\}$

(29) $\text{IE}(\text{Exh}_A\Diamond(p \vee q))(A') = A' - \emptyset = A'^{16}$

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    [15]Note also that the parse delivers the exclusion of $\Diamond r$ (the independent alternative) as a presupposition, contra the first finding in (24), though this prediction hinges on the particular choice of alternatives; if $\Diamond r$ were removed from A, and placed in A' instead of $\text{Exh}_A\Diamond r$, the exclusion of $\Diamond r$ would correctly result as an assertion. We return to this in Section 3.2.

    [16]The negations of the alternatives in A' do not jointly contradict the prejacent $\text{Exh}_A\Diamond(p \vee q)$; the negations amount to the proposition that p is permitted iff q is permitted, and if r is permitted then p or q is permitted. This is collectively consistent with $\Diamond(p \vee q)$, and so no subset of A' is subtracted in determining $\text{IE}(A')(\Diamond(p \vee q))$. Therefore $\text{IE}(A')(\Diamond(p \vee q)) = A' - \emptyset = A'$.

(30) $[\![(25\text{-a})]\!] = \lambda w : [\![\text{Exh}_A \lozenge (p \vee q))]\!](w){=}1$ (presupposition)

  $. \bigwedge\{[\![S']\!](w){=}0 : S' \in (29)\}$ (assertion)

  $= \lambda w : [\![\lozenge(p \vee q))]\!](w){=}1 \,\&\, [\![\underaccent{\dotuline}{\lozenge r}]\!]{=}0$

  $. [\![\text{Exh}_A \lozenge p]\!](w){=}0 \,\&\, [\![\text{Exh}_A \lozenge q]\!](w){=}0 \,\&\, \cancel{[\![\text{Exh}_A \lozenge r]\!](w){=}0}$

  $= \lambda w : [\![\lozenge(p \vee q)]\!](w){=}1 \,\&\, [\![\lozenge r]\!](w){=}0 . [\![\lozenge p]\!](w){=}1 \leftrightarrow [\![\lozenge q]\!](w){=}1$

  $= ([\![\lozenge p]\!] \,\&\, [\![\lozenge q]\!])_{([\![\lozenge(p \vee q)]\!] \,\&\, \neg[\![\lozenge r]\!])}$

If the empirical claims in Section 2 are correct, then (25-a) cannot be a possible parse for (15). What blocks it then? One possibility that I will discuss later, inspired by Gajewski and Sharvit (2012), is that (25-a) loses to another homophonous parse with stronger presuppositions. I will discuss this in Section 3.2, but because of space constraints I will not able to discuss it in the detail that it deserves.

Let us now turn to (25-b), where Exh appears above *only*. This parse correctly derives FC as a presupposition, but it does so only if presuppositions and exhaustification are assumed to interact in a particular way. In fact, the definitions of IE and Exh that we adopted allow exactly the needed interaction, as I will now show.

First, IE alternatives, given *S* and set *A*, are determined by removing any sets that minimally-cover *S*, i.e. any sets whose collective negations contradict *S*. But in (6)—where *cover* is defined—this is formulated more specifically: a set of alternatives *B* covers a sentence *S* iff the *falsity* of all its members is inconsistent with *S*. When those members carry presuppositions, their falsity requires that their presuppositions be met, since a sentence *S* that presupposes ϕ can only be false if its presupposition ϕ is true.

With this in mind, let's determine the IE-alternatives of $only_A \lozenge (p \vee q)_\text{F}$ in A':

(31) Let $S' = only_A \lozenge (p \vee q)$; and $A' = \{only_A \lozenge p,\ only_A \lozenge q,\ only_A \lozenge r\}$

On the assumption that $A = \{\lozenge p, \lozenge q, \lozenge r\}$, the prejacent S' presupposes $[\![\lozenge(p \vee q)]\!]$ and asserts that $[\![\lozenge r]\!]$ is false. The elements of A', *as well as their falsity*, presuppose (respectively) $[\![\lozenge p]\!]$, $[\![\lozenge q]\!]$, and $[\![\lozenge r]\!]$, each being the prejacent to *only* in the given alternative. Now, since S' asserts that $[\![\lozenge r]\!]$ is false, anything that presupposes $[\![\lozenge r]\!]$ will contradict it. This will therefore place $[only_A \lozenge r]$ in singleton that covers S', because its falsity alone is inconsistent with S'. The remaining two members of A' do not cover S', because their negations presuppose $[\![\lozenge p]\!]$ and $[\![\lozenge q]\!]$—so far consistently with S'—and assert $([\![\lozenge q]\!] \vee [\![\lozenge r]\!])$ and $([\![\lozenge p]\!] \vee [\![\lozenge r]\!])$, also consistently with S'. Therefore,

(32) $\text{IE}(only_A \lozenge (p \vee q))(A') = A' - \{only_A \lozenge r\} = \{only_A \lozenge p,\ only_A \lozenge q\}$

Next we turn to our definition of Exh:

(4) $[\![\text{Exh}_A\, S]\!] = \lambda w . [\![S]\!](w){=}1 \,\&\, \bigwedge\{\underline{[\![S']\!](w){=}0} : S' \in \text{IE}(S)(A)\}$

According to (4), $[\![\text{Exh}_A\, S]\!]$ is true only if the IE-alternatives of *S* are false. But these IE alternatives can only be false if their presuppositions are met, otherwise they would be undefined. Exh is therefore defined only in worlds where the presuppositions of the relevant

IE alternatives are true. This gives us FC as a presupposition for parse (25-b), as the details in (33) show.

(33) $[\![\mathrm{Exh}_{A'}(only_A(p\vee q)_{\mathrm{F}})]\!] = \lambda w\,.\,[\![only_A(p\vee q)]\!](w)\!=\!1\,\&\,\bigwedge\{[\![S'']\!]\!=\!0 : S'' \in (32)\}$

$= \lambda w\,.\,[\![only_A(p\vee q)]\!](w)\!=\!1\,\&\,\boxed{\mathrm{A}}(w)\!=\!0\,\&\,\boxed{\mathrm{B}}(w)\!=\!0$

$= (\neg[\![\lozenge r]\!])_{[\![\lozenge(p\vee q)]\!]}\,\&\,([\![\lozenge q]\!]\vee[\![\lozenge r]\!])_{[\![\lozenge p]\!]}\,\&\,([\![\lozenge p]\!]\vee[\![\lozenge r]\!])_{[\![\lozenge q]\!]}$

$\boxed{= (\neg[\![\lozenge r]\!])_{[\![\lozenge p]\!]\&[\![\lozenge q]\!]}}$

$\boxed{\mathrm{A}}\quad [\![only_A\lozenge(p)_{\mathrm{F}}]\!] = (\neg[\![\lozenge q]\!]\&\neg[\![\lozenge r]\!])_{[\![\lozenge p]\!]}$

$\boxed{\mathrm{B}}\quad [\![only_A\lozenge(q)_{\mathrm{F}}]\!] = (\neg[\![\lozenge p]\!]\&\neg[\![\lozenge r]\!])_{[\![\lozenge q]\!]}$

Exhaustification above *only*, then, correctly derives FC as a presupposition, and the exclusion of $\lozenge r$ as part of the assertive meaning. But as we just saw, this requires treating Exh as a *presupposition hole*.

The idea of defining Exh as a presupposition hole was recently defended in Spector and Sudo (2014). The example used by Spector and Sudo is shown in (34).

(34)   John is aware that some$_{(\mathrm{F})}$ of the students smoke.   ($aware_j(\exists)$)

Spector and Sudo argue that (34) can be uttered truthfully in contexts where all of the students smoke, but John believes that some of them do, and is open to the possibility that they all do (the reading is helped with prosodic prominence on *some*). If the judgement is accurate, then the embedded clause cannot contain an Exh operator in it, for otherwise we would understand John's belief to be that some *and not all* students smoke. The presupposition that all of them smoke, which is easily detected with prominence on *some*, must then be generated from matrix-level exhaustification, and this is possible if Exh is defined as a presupposition hole.

(35) $A = \{aware_j(\forall)\}$

(36) $[\![\mathrm{Exh}_A(aware_j(\exists))]\!] = [\![aware_j(\exists)]\!]\,\&\,\neg[\![aware_j(\forall)]\!]$

$= [\![believe_j(\exists)]\!]_{[\![\exists]\!]}\,\&\,(\neg[\![believe_j(\forall)]\!])_{[\![\forall]\!]}$

3.1   The exclusive inference

It was argued in Section 2 that the exclusive inference $\neg\lozenge(p\wedge q)$ is neither presupposed nor asserted by sentence (15). For parse (25-b), repeated again, this requires adding the conjunctive alternative to A', the Exh alternative set, and crucially leaving it out of A, where it would otherwise generate the inference in the assertive content, contra the claim in (24).

(25-b) $\mathrm{Exh}_{A'}(only_A\lozenge(p\vee q)_{\mathrm{F}})$

I cannot think of good reasons why $\lozenge(p\wedge q)$ should be absent from A. Perhaps its presence requires accenting *or* in *only*'s prejacent, but where might this requirement come from? By assumption, the entire disjunction is focused, and so its scalar contents should be subject to replacement, and should therefore produce the conjunctive alternative in A. One might

think that, if the same could be achieved with narrower focus, that is, if narrower focus in the prejacent (e.g. by accenting *or* and nothing else) can place the conjunctive alternative in A, then narrower focus is preferred.[17] But narrowing the focus in this case will fail to admit $\Diamond r$ in A, since $\Diamond r$ is not an alternative to the accented connective. To keep $\Diamond r$ in A, then, broad focus is necessary. Why this makes it difficult to replace *or* with *and* within the focused constituent is a question that I leave for future work.

3.2   Double-exhaustification under *only*?

So far we've only considered parses of (15) where *only* is accompanied by a single Exh operator. But there is another possible parse in which *only* embeds two Exhs (37):

(37) $only_{A''}(\text{Exh}_{A'}\text{Exh}_A\Diamond(p \vee q)_\text{F}))$

Since double-exhaustification derives FC as part of its meaning, and since in (37) *only* takes (and hence presupposes) a prejacent containing the two Exhs, (37) will also presuppose FC.

   The question now is whether (37) can correctly deliver $\neg\Diamond r$ as part of its assertive content. The answer depends on how alternatives are distributed among A, A', A''. First, neither A nor A' can contain $\Diamond r$. If they did, its negation will be part of the embedded exhaustification, and will therefore be presupposed by *only*, contra the first finding of (24). Second A'' must contain $\Diamond r$, and must not contain $[\text{Exh}_{A'}\text{Exh}_A\Diamond r]$. But what guarantees this? After all, the alternatives to *only* are formed by replacing the *focused elements* in its prejacent with their alternatives. In this case the focused element is the disjunction $(p \vee q)$, whose alternatives are assumed to contain p and q, and plausibly r if it is contextually salient. But replacing just the (focused) disjunction with r will generate $[\text{Exh}_{A'}\text{Exh}_A\Diamond r]$ as an alternative, and negating this alternative will not deliver the correct result.[18]

$$A = \{\Diamond p,\ \Diamond q\}, A' = \{\text{Exh}_A\Diamond p,\ \text{Exh}_A\Diamond q\},$$
$$A'' = \{\text{Exh}_{A'}(\text{Exh}_A\Diamond p),\ \text{Exh}_{A'}(\text{Exh}_A\Diamond q),\ \text{Exh}_{A'}(\text{Exh}_A\Diamond r)\}$$
$$(\equiv \Diamond p \wedge \neg\Diamond q)\quad (\equiv \Diamond q \wedge \neg\Diamond p)\quad (\equiv \Diamond r \wedge \neg\Diamond(p \vee q))$$

So, in order for (37) to work, A'' must contain $\Diamond r$ rather than its (doubly)-exhaustified parse. And indeed there is some justification for this. Recently Gajewski and Sharvit (2012) argued that in Strawson-DE environments, exhaustification is detected in the (upward-entailing) presupposition, but is absent from the (downward-entailing) assertive meaning. Gajewski and Sharvit use negative factive predicates like *be sorry that* to make their point, but here we may replicate their findings using *only*, whose prejacent (minus the focus) is also SDE.[19] Consider (38), uttered without prosodic prominence on *or*.

(38)   Only John$_\text{F}$ talked to Mary or Sue.

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[17]This would be a version of Schwarzschild's (1999) AvoidF constraint. See also Fox and Spector (2013).

[18]The falsity of $\text{Exh}_{A'}(\text{Exh}_A\Diamond(p \vee q))$ follows from the prejacent, so it will not contribute anything. The alternative is true iff r is allowed and neither p nor q is, and so its negation is the proposition that if r is allowed, then p or q is. But the consequent of this conditional is already entailed by the prejacent.

[19]See von Fintel (1999) and Wagner (2006).

By default, (38) suggests that no one other than John talked to *either of Mary or Sue*. At the same time, the sentence suggests that John talked to one of them, but not to both. The latter part results if we assume an embedded Exh operator under *only*:

(39)   Only John$_F$ Exh(talked to Mary or Sue)

But if *only*'s assertoric component negated alternatives that also contain Exh, we predict the sentence to be true if John's alternatives (e.g. Bill, Fred, etc) talked to *both* Mary and Sue, since speaking to both of them would indeed make the exhaustified alternative false.

   In light of this behavior, Gajewski and Sharvit propose a multi-tiered semantic model that computes presuppositions and assertions separately, and in addition *strengthens* them separately[20]: if the prejacent is parsed with Exh, this will make *only*'s presupposition stronger, but its assertion *weaker*, since ⟦Exh(John talked to Mary or Sue)⟧ asymmetrically entails ⟦John talked to Mary or Sue⟧, but ¬⟦Exh(Bill talked to Mary or Sue)⟧ is asymmetrically *entailed by* ¬⟦Bill talked to Mary or Sue⟧. And so the decision of whether or not to exhaustify, according to Gajewski and Sharvit, is made separately in the two semantic tiers, and within the given tier the choice depends on whether exhaustification results in a stronger outcome.

   If this idea is applied to (37), then it *follows* that $\Diamond r$, rather than its doubly-exhaustified parse, should be negated by the assertion of *only*, since the assertion will be stronger without exhaustification. By contrast, the presuppositional component will be stronger *with* exhaustification, and in this case, it will be even stronger if it is *doubly*-exhaustified. This brings us back to why parse (25-a) is blocked:

(25-a) $only_{A'}(\text{Exh}_A \Diamond (p \vee q)_F)$

On a system like Gajewski and Sharvit's, (25-a) gives rise to a presupposition that could have been interpreted more strongly, because further exhaustification would have generated FC, rather than the weaker $\Diamond(p \vee q)$, in the presuppositional tier.

   These are sketches of how FC can be generated as a presupposition within the scope of *only*. There is independent motivation, it seems, for the idea that exhaustification can apply to presuppositions, and moreover, that it can apply to them without applying to the assertive contribution of the same sentence. This narrows down the possible ways of parsing (15), and explains why the independent alternative $\Diamond r$ is negated without exhaustification. What remains to be seen is whether the exclusive inference be be plausibly attributed to a yet higher occurrence of Exh.

4.   Conclusion

We began with the observation that Fox's theory of FC delivers three inferences via exhaustification: the exclusive inference, FC, and the negation of independent alternatives. We then saw that these inferences show different behavior when an FC-permission sentence is embedded under *only*: the independent alternatives are negated as an assertion, the FC inference behaves like a presupposition, and the exclusive inference behaves like

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[20]In assuming separate tiers Gajewski and Sharvit follow Karttunen and Peters (1979).

a scalar implicature. We considered accounting for this behavior by substituting *only* for one of the two Exh operators (in Fox's LF), and saw that exhaustifying above *only* makes more accurate predictions, as long as Exh is assumed to be a presupposition hole. This assumption was motivated with arguments that were recently made by Spector and Sudo (2014). We were left with the task of independently blocking the incorrect parse (where Exh appears below *only*), and cited findings from Gajewski and Sharvit (2012) that elect yet another parse—one containing *two* Exhs below *only*—which blocks the undesired LF, and also generates the inferences correctly. The task of distinguishing the two parses is left for another occasion.

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